Homework 1 - due January 20th

1. It is 2032 and you are a professor at the Institute for Writing And Research Mathematics (WARM). After class at WARM one day, one of your students asks you to look over their "proof" for the problem

> You have a white box with 60 white balls and a black box with 60 black balls. You take 20 balls from the white box, put them in the black box, mix the balls around and then randomly take out 20 balls and put them in the white box. In the end, which is larger: the number of black balls in the white box, or the number of white balls in the black box?

Proof. After we move 20 white balls to the black box, if we then move 1 black ball and 19 white balls back to the white box, then 1 white ball was left in the black box. This proves the base case for our induction. Now for our induction step let there be 61 balls in each box. Then the same thing will happen by probability.

Therefore the numbers are the same because the number of black balls in the white box equals the number of white balls in the black box. □

(a) In which of the following ways is this not a proof? Explain your answer.

- It has parts that are too vague to be a rigorous proof.
- It uses irrelevant concepts.
- It restates the claim we are tying to prove using it as an explanation.
- It is an example.

(b) Prove that for every *odd* positive integer n, we have $n^2 - 1$ is divisible by 8.

Homework 2 - due January 27th

2. One day in the coffee room, one of your fellow professors at WARM comes up to you and says "All positive integers are even! I have a proof!"

What is wrong with the following proof that they give you?

Proof. We are going to do a strong induction on the positive integers.
Assume that every positive integer up to and including n is even. Then we want to show that $n+1$ is even. By the induction hypothesis we know that $n-1$ is even. Therefore $n-1 = 2m$ for some integer $m > 0$. Hence $n+1 = n-1+2 = 2m+2 = 2(m+1)$ is even, and the result follows by induction. □

Homework 3 - due February 3rd

3. For LaTeX practice, transcribe all of your research journals so far into *one long document* in LaTeXto hand in. Do the following.

- Each weekly entry should start a new page. It should say which journal entry it is as a title. Your name and student number should also be on this page.
- There should be three sections with headings within each entry: What I did; Why I did it; What obstacles I encountered and my research plan for the following week.
- Make sure it is in 12 point font.
- ***From now on:*** Please latex up all submissions including your homework and your journal entry. In all cases submit the PDF created. Thank you.

Homework 4 - due February 10th

4. Interview one of your UBC mathematics instructors (past or present, but not me as we have talked about this together already) for about 15 minutes. You can do this over Zoom, in person, or over email.

If you do Zoom or in person then I recommend that you agree a time (for example during their office hours) and if you do email then I recommend that you get them to agree to send you an answer well before the homework deadline (for example on the Monday).

Ask them the following.

(a) Where do they get ideas for a new mathematics research problem to work on, and how do they start working on it?

(b) What do they do if they get stuck?

Submit for your homework a 1-2 page write-up in LaTeX of who you interviewed, and what did they answer for each question.

Homework 5 - due February 17th

5. *Beamer presentations:* Using the two downloadable files about Beamer, as a resource, make 3 Beamer slides about a hobby or interest of yours.

- Make a title page *before* the 3 slides with your hobby/interest as a title, and your name.
- Have at least one `pause` in one of the 3 slides.
- Upload the PDF of your slides to Canvas.

Homework 6 - due March 10th

6. Write a short biography of 200-300 words on one of the following professors:

Julia Gordon, Cindy Greenwood, Leah Keshet, Costanza Piccolo, Malabika Pramanik, Elina Robeva, Sujatha Ramdorai, Steph van Willigenburg, Elyse Yeager.

The biography should include their

- name,
- current position at UBC,
- upbringing and education and positions before UBC,
- mathematical interests, and
- notable career achievements in research/teaching/service to the community such as prizes.

You may also include a fun fact about them.

You can obtain the information via an interview in person or email giving the date it happened, or other sources such as websites that you must cite.

Below the biography state the word count to make sure that you have not written too much or too little. You will be graded on content and writing.

Asap: Please let me know by email the name of the professor you would like to write on. Names will be allocated on a first come first served basis, with no more than 2 students assigned to any one professor. If you have not emailed me and then received an allocation, the homework will not count.

Note: The best biographies graded by content and writing quality will be turned into posters by the Mathematics Department Equity, Diversity, and Inclusion Committee to be put on display in the Math Department and MLC!

Homework 7 - due March 17th

7. One of your colleagues at WARM tells you about an old theorem they read about:

If $k+1$ or more objects are put into k boxes,
then at least one box contains more than one object.

Use this to prove the following.

- Let m be any positive integer. Prove that if you are given 11 distinct positive powers of m, then there exists two of them, m^a and m^b, such that $m^a - m^b$ is divisible by 10.

Homework 8 - due March 24th

8. Download a copy of "Polygon Dissections and Standard Young Tableaux" and

- Explain **four** ways in which the local structure of this paper could be improved.
- Place your four explanations as four items in a list.

Homework 9 - due March 31st

9. Twelve sticks are laid out as you see them below. Perform the following operations:

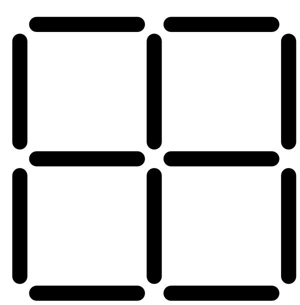
(a) remove 2 sticks to obtain 2 unequal squares

(b) move 3 sticks to obtain 3 equal squares

(c) move 4 sticks to obtain 3 equal squares

(d) move 2 sticks to obtain 7 squares

(e) move 4 sticks to obtain 10 squares.



In all cases all remaining sticks must contribute to at least one square.

Homework 10 - due April 7th

10. *Final reflections:* In Math 444 you have learned new mathematics and many techniques needed to become a research mathematician (proof techniques, writing techniques, LaTeX, presentation skills, team work etc).

What are the *three* most valuable things you have learned, and why?

Note: This will also count as your Journal 10.