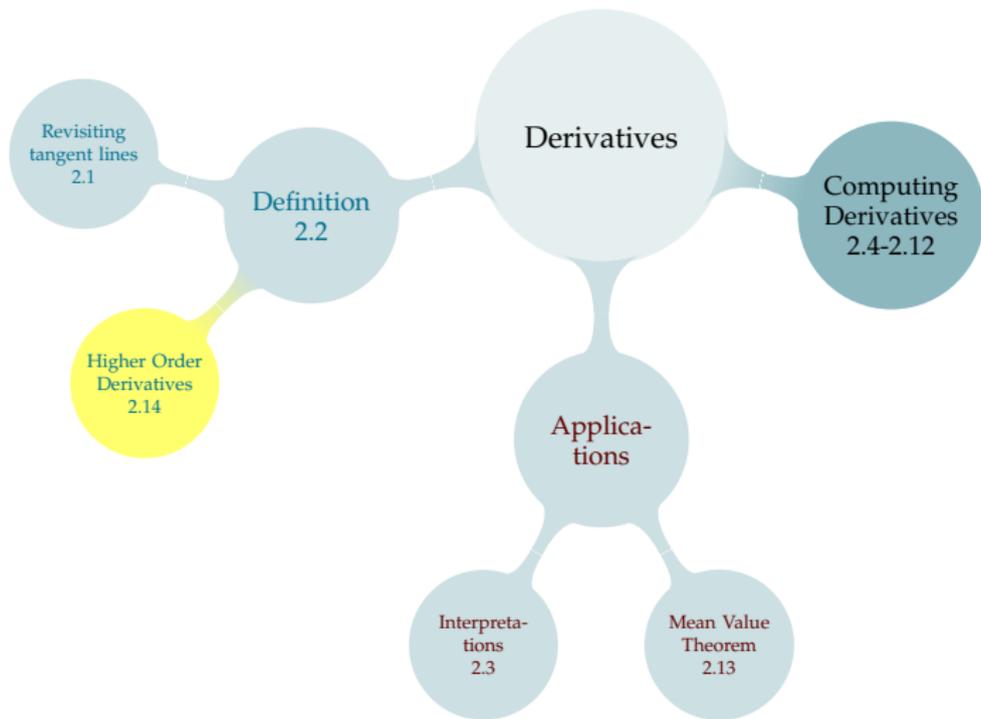


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HIGHER ORDER DERIVATIVES

Evaluate $\frac{d}{dx} \left[\frac{d}{dx} [x^5 - 2x^2 + 3] \right]$

$$\frac{d}{dx} [x^5 - 2x^2 + 3] =$$

Notation 2.14.1

The derivative of a derivative is called the **second derivative**, written

$$f''(x) \quad \text{or} \quad \frac{d^2y}{dx^2}(x)$$

Similarly, the derivative of a second derivative is a third derivative, etc.

Notation 2.14.1

- ▶ $f''(x)$ and $f^{(2)}(x)$ and $\frac{d^2f}{dx^2}(x)$ all mean $\frac{d}{dx} \left(\frac{d}{dx} f(x) \right)$
- ▶ $f'''(x)$ and $f^{(3)}(x)$ and $\frac{d^3f}{dx^3}(x)$ all mean $\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right)$
- ▶ $f^{(4)}(x)$ and $\frac{d^4f}{dx^4}(x)$ both mean $\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right)$
- ▶ and so on.

TYPICAL EXAMPLE: ACCELERATION

- ▶ Velocity: rate of change of position
- ▶ Acceleration: rate of change of velocity.

The position of an object at time t is given by $s(t) = t(5 - t)$. *Time is measured in seconds, and position is measured in metres.*

1. Sketch the graph giving the position of the object.
2. What is the velocity of the object when $t = 1$? Include units.
3. What is the acceleration of the object when $t = 1$? Include units.

CONCEPT CHECK

True or False: If $f'(1) = 18$, then $f''(1) = 0$, since the $\frac{d}{dx}\{18\} = 0$.

Which of the following is always true of a QUADRATIC polynomial $f(x)$?

- A. $f(0) = 0$
- B. $f'(0) = 0$
- C. $f''(0) = 0$
- D. $f'''(0) = 0$
- E. $f^{(4)}(0) = 0$

Which of the following is always true of a CUBIC polynomial $f(x)$?

- A. $f(0) = 0$
- B. $f'(0) = 0$
- C. $f''(0) = 0$
- D. $f'''(0) = 0$
- E. $f^{(4)}(0) = 0$

IMPLICIT DIFFERENTIATION

Suppose $y(x)$ is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find $y''(x)$ at the point $(-2, 1)$.

Included Work