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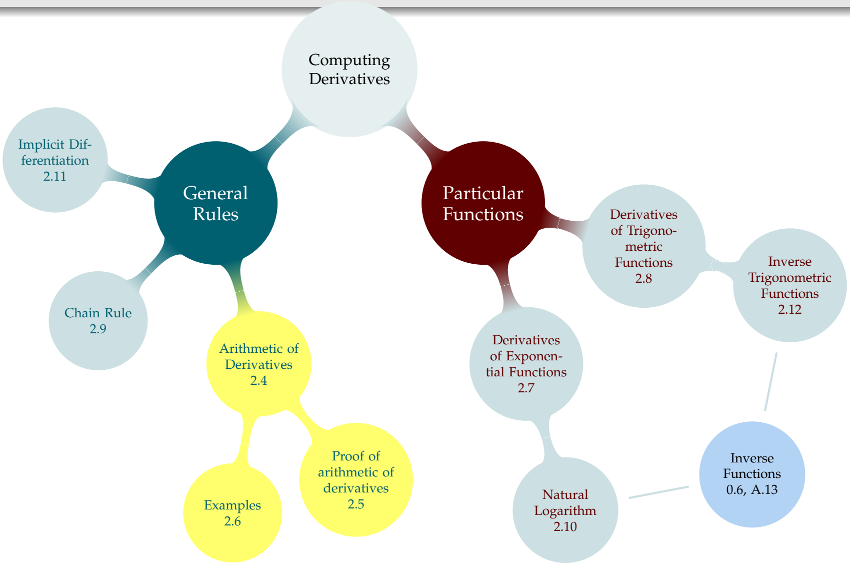

Derivatives of Lines

$$f(x) = 2x - 15$$

The equation of the tangent line to $f(x)$ at $x = 100$ is:

$f'(1) =$
A. 0 B. 1 C. 2 D. -15 E. -13

$f'(5) =$

$f'(-13) =$

$$g(x) = 13$$

$g'(1) =$

A. 0 B. 1 C. 2 D. 13

Adding a Constant

Adding or subtracting a constant to a function does not change its derivative.

We saw

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(3 - 0.8t^2\right)\bigg|_{t=1} = -1.6$$

So,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(10 - 0.8t^2\right)\bigg|_{t=1} =$$

DIFFERENTIATING SUMS

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{f(x) + g(x)\right\} =$$

Constant Multiple of a Function

Let a be a constant.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{a \cdot f(x)\right\} =$$

Rules – Lemma 2.4.1

Suppose $f(x)$ and $g(x)$ are differentiable, and let c be a constant number. Then:

- $\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)+g(x)\} = f'(x)+g'(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)-g(x)\} = f'(x)-g'(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\{cf(x)\} = cf'(x)$

For instance: let $f(x) = 10\left((2x-15)+13-\sqrt{x}\right)$. Then $f'(x) =$

NOW YOU

Suppose $f'(x) = 3x$, $g'(x) = -x^2$, and $h'(x) = 5$.

Calculate:

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(x) + 5g(x) - h(x) + 22\}$$

A. $3x - 5x^2$

B. $3x - 5x^2 - 5$

C. $3x - 5x^2 - 5 + 22$

D. none of the above

Derivatives of Products

$$\frac{d}{dx}\{x\} = 1$$

True or False:

$$\begin{aligned}\frac{d}{dx}\{2x\} &= \frac{d}{dx}\{x+x\}\\ &= [1]+[1]\\ &= 2\end{aligned}$$

True or False:

$$\begin{aligned}\frac{d}{dx}\{x^2\} &= \frac{d}{dx}\{x\cdot x\}\\ &= [1]\cdot[1]\\ &= 1\end{aligned}$$

What to do with Products?

Suppose $f(x)$ and $g(x)$ are differentiable functions of x. What about $f(x)g(x)$?

Product Rule – Theorem 2.4.3

For differentiable functions $f(x)$ and $g(x)$:

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Example:

$$\frac{d}{dx}\left[x^2\right] =$$

Example: suppose $f(x) = 3x^2, f'(x) = 6x$, $g(x) = \sin(x), g'(x) = \cos(x).$

$$\frac{d}{dx}\left[3x^2 \sin(x)\right] =$$

Given $\frac{d}{dx}[2x+5] = 2, \quad \frac{d}{dx}\left[\sin(x^2)\right] = 2x\cos(x^2), \quad \frac{d}{dx}\left[x^2\right] = 2x$

Now You

$$f(x) = (2x+5)\sin(x^2)$$

A. $f'(x) = (2)\left(2x\cos(x^2)\right)(2x)$
B. $f'(x) = (2)\left(2x\cos(x^2)\right)$
C. $f'(x) = (2x+5)(2) + \sin(x^2)\left(2x\cos(x^2)\right)$
D. $f'(x) = (2x+5)\left(2x\cos(x^2)\right) + (2)\sin(x^2)$
E. none of the above

Now You

$$f(x) = a(x) \cdot b(x) \cdot c(x)$$

What is $f'(x)$?

Quotient Rule – Theorem 2.4.5

Let $f(x)$ and $g(x)$ be differentiable and $g(x) \neq 0$. Then:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

Mnemonic: Low d'high minus high d'low over lowlow.

Quotient Rule – Theorem 2.4.5

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{2x+5}{3x-6}\right\} =$$

Quotient Rule – Theorem 2.4.5

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Mnemonic: Low d'high minus high d'low over lowlow.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{5x}{\sqrt{x}-1}\right\} =$$

Now You

Differentiate the following.

$$f(x) = 2x + 5$$

$$g(x) = (2x+5)(3x-7) + 25$$

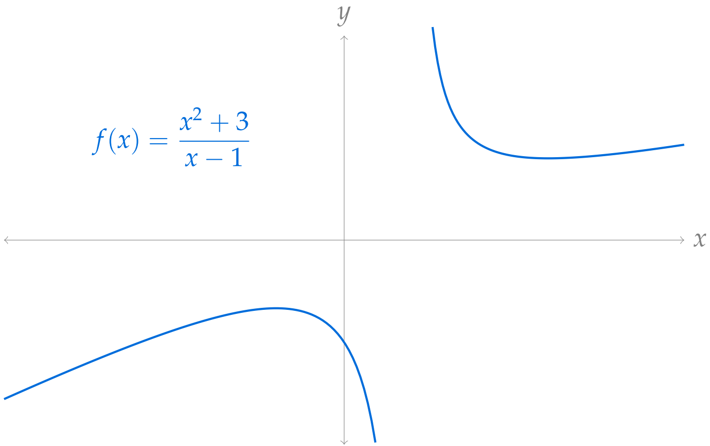
$$h(x) = \frac{2x+5}{8x-2}$$

$$j(x) = \left(\frac{2x+5}{8x-2}\right)^2$$

Rules

Product: $\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)g(x)\} = f(x)g'(x) + g(x)f'(x)$

Quotient: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\dfrac{f(x)}{g(x)}\right\} = \dfrac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

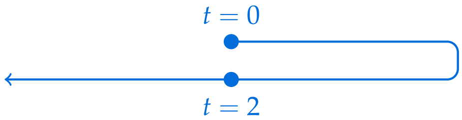


For which values of x is the tangent line to the curve horizontal?

The position of an object moving left and right at time t, $t \geq 0$, is given by

$$s(t) = -t^2(t-2)$$

where a positive position means it is to the right of its starting position, and a negative position means it is to the left. First it moves to the right, then it moves left forever.



What is the farthest point to the right that the object reaches?

More About the Product Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\{x^2\} = \frac{\mathrm{d}}{\mathrm{d}x}\{x \cdot x\} = x(1) + x(1)$$
$$= 2x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{x^3\} = \frac{\mathrm{d}}{\mathrm{d}x}\{x \cdot x^2\}$$
$$= (x)(2x) + (x^2)(1) = 3x^2$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{x^4\} = \frac{\mathrm{d}}{\mathrm{d}x}\{x \cdot x^3\}$$
$$= x(3x^2) + x^3(1) = 4x^3$$

Where are these functions defined?

function	derivative
x	1
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^{30}	$30x^{29}$
x^n	nx^{n-1}

Cautionary Tale

WITH *functions* RAISED TO A POWER, IT'S MORE COMPLICATED.

Differentiate $(2x+1)^2$

Power Rule – Corollary 2.6.17

$\frac{\mathrm{d}}{\mathrm{d}x}\{x^a\} = ax^{a-1}$ (where defined)

$$\frac{\mathrm{d}}{\mathrm{d}x}\{3x^5 + 7x^2 - x + 15\} =$$

Power Rule – Corollary 2.6.17

$\frac{\mathrm{d}}{\mathrm{d}x}\{x^a\} = ax^{a-1}$ (where defined)

Differentiate $\dfrac{(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})}{2x+5}$

Power Rule – Corollary 2.6.17

$\frac{\mathrm{d}}{\mathrm{d}x}\{x^a\} = ax^{a-1}$ (where defined)

Suppose a motorist is driving their car, and their position is given by $s(t) = 10t^3 - 90t^2 + 180t$ kilometres. At $t = 1$ (t measured in hours), a police officer notices they are driving erratically. The motorist claims to have simply suffered a lack of attention: they were in the act of pressing the brakes even as the officer noticed their speed.

At $t = 1$, how fast was the motorist going, and were they pressing the gas or the brake?

Challenge: What about $t = 2$?

Power Rule – Corollary 2.6.17

$\frac{\mathrm{d}}{\mathrm{d}x}\{x^a\} = ax^{a-1}$ (where defined)

Recall that a sphere of radius r has volume $V = \frac{4}{3}\pi r^3$.
Suppose you are winding twine into a gigantic twine ball, filming the process, and trying to make a viral video. You can wrap one cubic meter of twine per hour. (In other words, when we have V cubic meters of twine, we're at time V hours.) How fast is the radius of your spherical twine ball increasing?

Included Work