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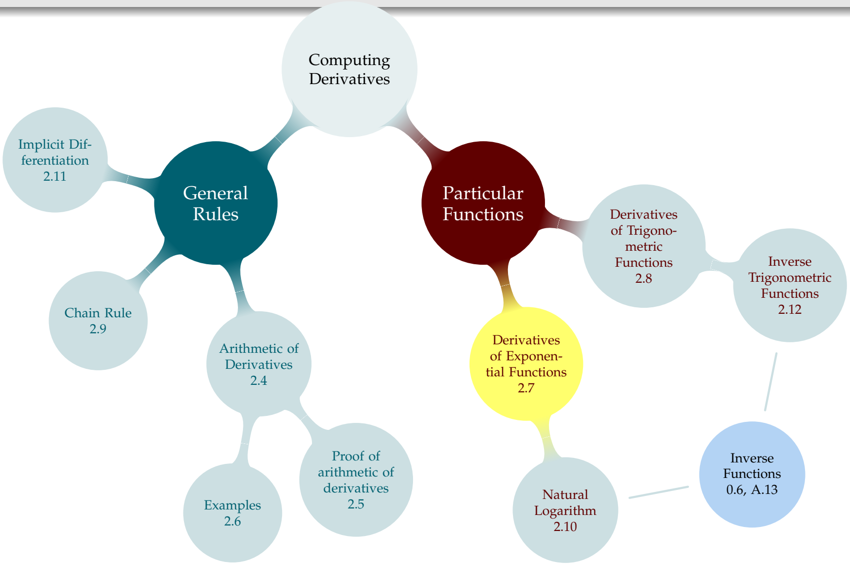
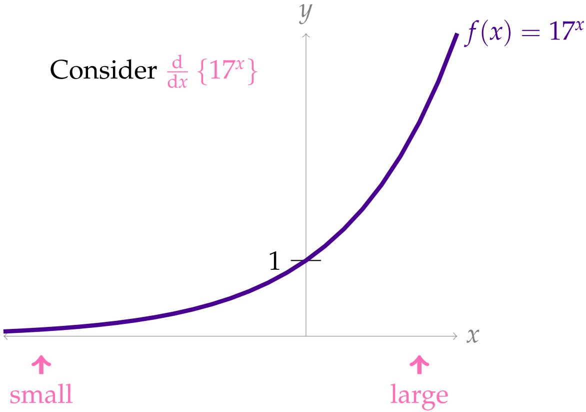


Exponential Functions

Consider $\frac{\mathrm{d}}{\mathrm{d}x}\left\{17^x\right\}$



$f(x)$ is always increasing, so $f'(x)$ is always positive.
$f'(x)$ might look similar to $f(x)$.

Exponential Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that $\displaystyle\lim_{h\to 0}\frac{17^h - 1}{h} = 0$?

A. Sure, there's no reason we've seen that would make it impossible.

B. No, it couldn't be 0, that wouldn't make sense.

C. I do not feel equipped to answer this question.

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

Given what you know about $\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\}$, **is it possible** that $\displaystyle\lim_{h\to 0}\frac{17^h - 1}{h} = \infty$?

A. Sure, there's no reason we've seen that would make it impossible.

B. No, it couldn't be ∞, that wouldn't make sense.

C. I do not feel equipped to answer this question.

$$\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} = 17^x \cdot \underbrace{\lim_{h\to 0}\frac{(17^h - 1)}{h}}_{\text{constant}}$$

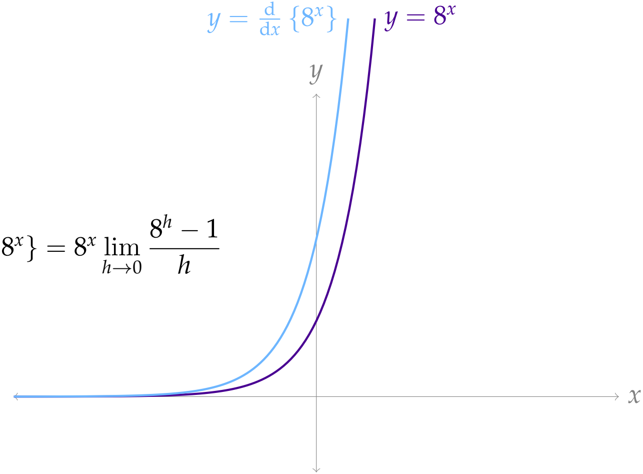
h	$\dfrac{17^h - 1}{h}$
0.001	2.83723068608
0.00001	2.83325347992
0.0000001	2.83321374583
0.000000001	2.83321344163

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\{17^x\} &= \lim_{h\to 0}\frac{17^{x+h}-17^x}{h}\\ &= \lim_{h\to 0}\frac{17^x 17^h-17^x}{h}\\ &= \lim_{h\to 0}\frac{17^x(17^h-1)}{h}\\ &= 17^x\lim_{h\to 0}\frac{(17^h-1)}{h}\end{aligned}$$

In general, for any positive number a,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x\lim_{h\to 0}\frac{a^h-1}{h}$$

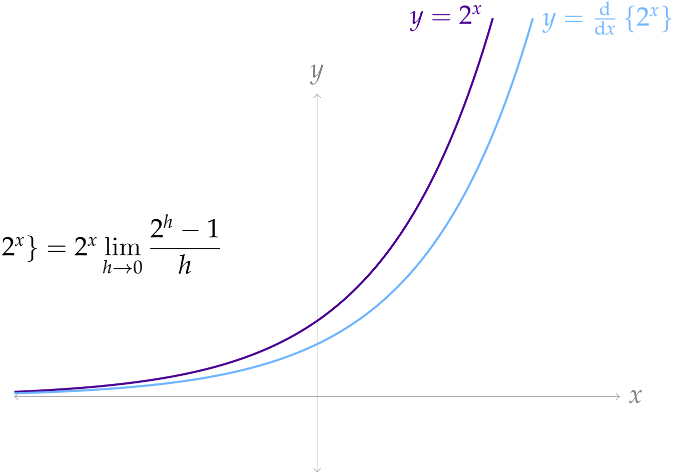
Exponential Functions



$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{8^x\right\} = 8^x \lim_{h \to 0} \frac{8^h - 1}{h}$$

Exponential Functions

$$\frac{d}{dx}\left\{2^x\right\} = 2^x \lim_{h\to 0} \frac{2^h - 1}{h}$$



In general, for any positive number a, $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \lim_{h\to 0} \frac{a^h - 1}{h}$

Euler's Number – Theorem 2.7.4

We define e to be the unique number satisfying

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$e \approx 2.7182818284590452353602874713526624\ldots$ (Wikipedia)

Theorem 2.7.4 and Corollary 2.10.6

Using this definition of e,

$$\frac{\mathrm{d}}{\mathrm{d}x}\{e^x\} = e^x \underbrace{\lim_{h\to 0}\frac{e^h - 1}{h}}_{1} = e^x$$

In general, $\lim_{h\to 0}\frac{a^h - 1}{h} = \log_e(a)$, so $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \log_e(a)$

That $\lim_{h\to 0}\frac{a^h - 1}{h} = \log_e(a)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\{a^x\} = a^x \log_e(a)$ are consequences of

$$a^x = \left(e^{\log_e(a)}\right)^x = e^{x\log_e(a)}$$

For the details, see the end of Section 2.7.

Things to Have Memorized

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^x\right\} = e^x$$

When a is any constant,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{a^x\right\} = a^x \log_e(a)$$

Let $f(x) = \dfrac{e^x}{3x^5}$. When is the tangent line to $f(x)$ horizontal?

Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{3x}\right\}$

Suppose the deficit, in millions, of a fictitious country is given by

$$f(x) = e^x(4x^3 - 12x^2 + 14x - 4)$$

where x is the number of years since the current leader took office. Suppose the leader has been in power for exactly two years.

1. Is the deficit increasing or decreasing?

2. Is the rate at which the deficit is growing increasing or decreasing?