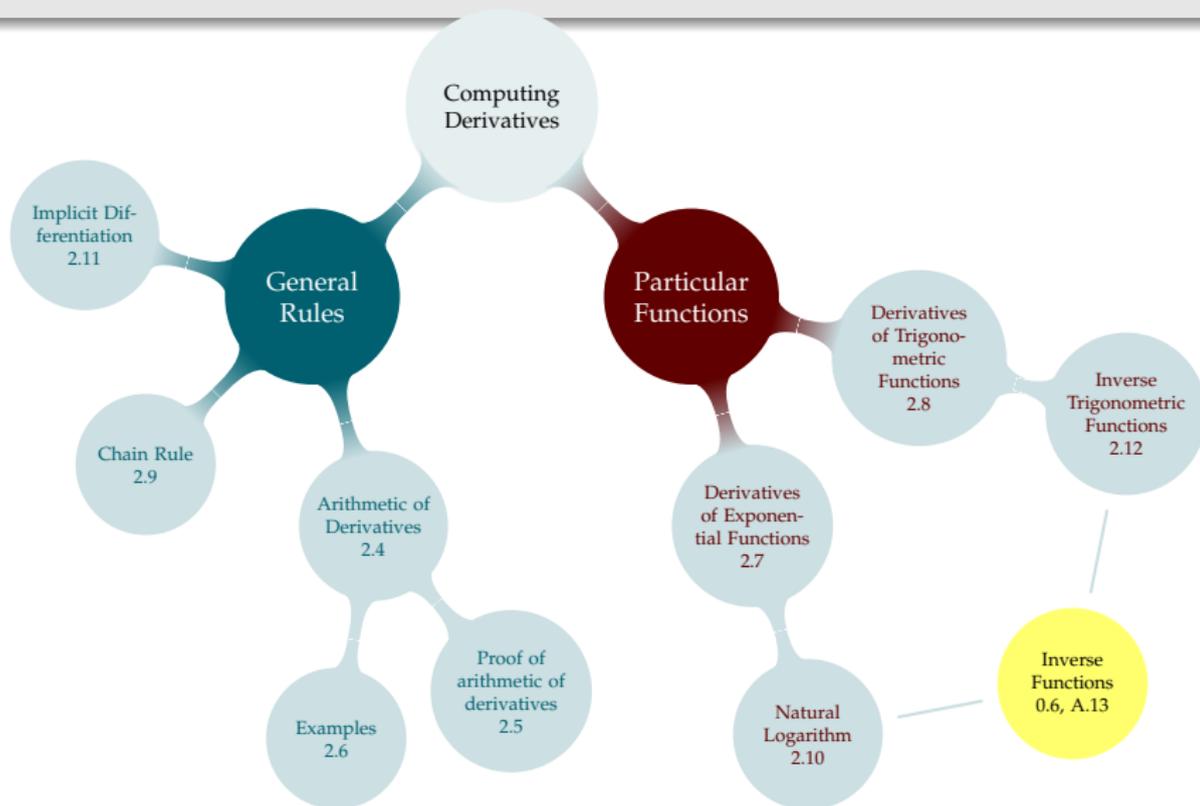


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INVERTIBILITY GAME

- ▶ A function $y = f(x)$ is known to both players
- ▶ **Player A** chooses a secret value x in the domain of $f(x)$
- ▶ **Player A** tells **Player B** what $f(x)$ is
- ▶ **Player B** tries to guess **Player A**'s x -value.

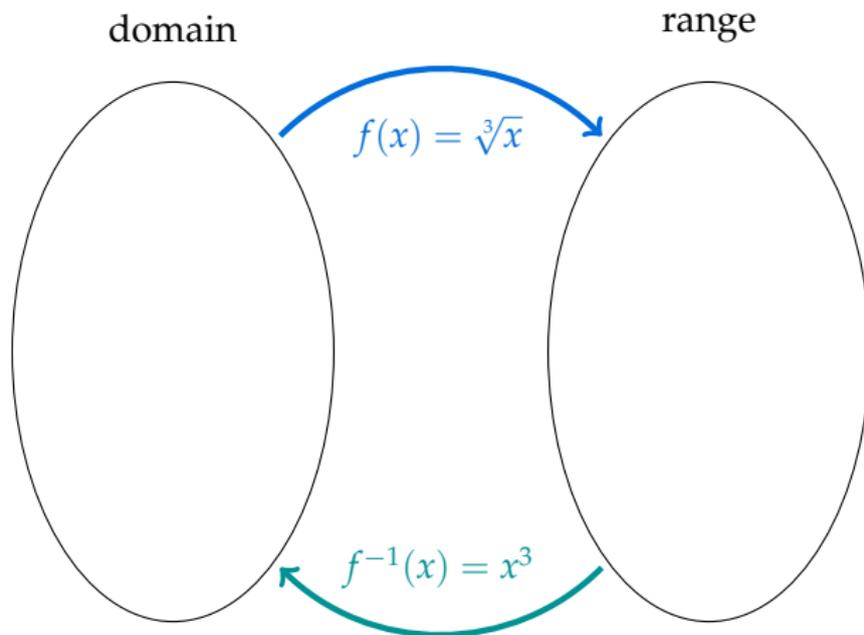
Round 1: $f(x) = 2x$

Round 2: $f(x) = \sqrt[3]{x}$

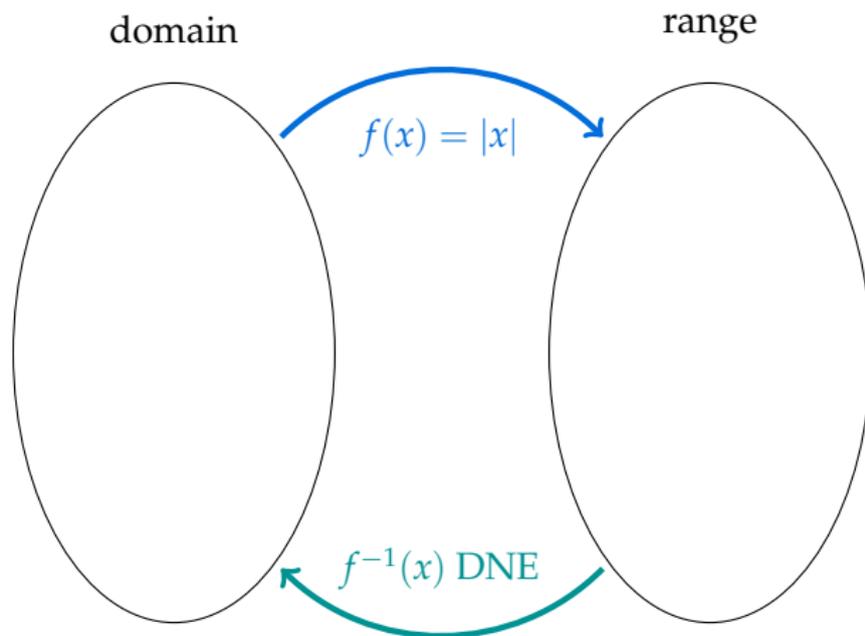
Round 3: $f(x) = |x|$

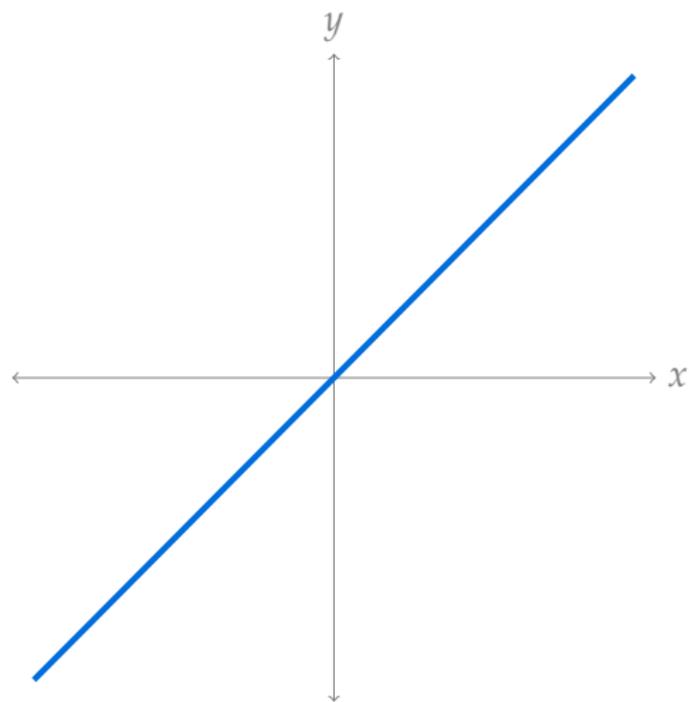
Round 4: $f(x) = \sin x$

FUNCTIONS ARE MAPS



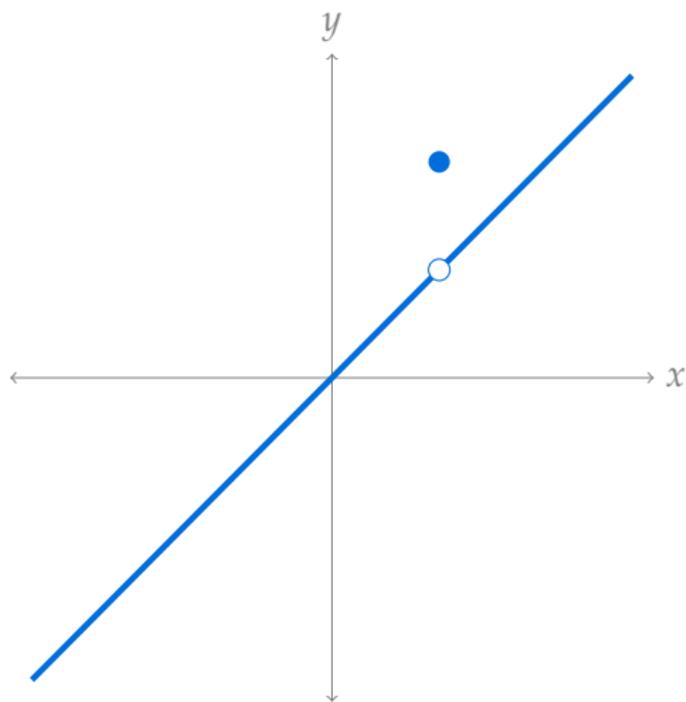
FUNCTIONS ARE MAPS





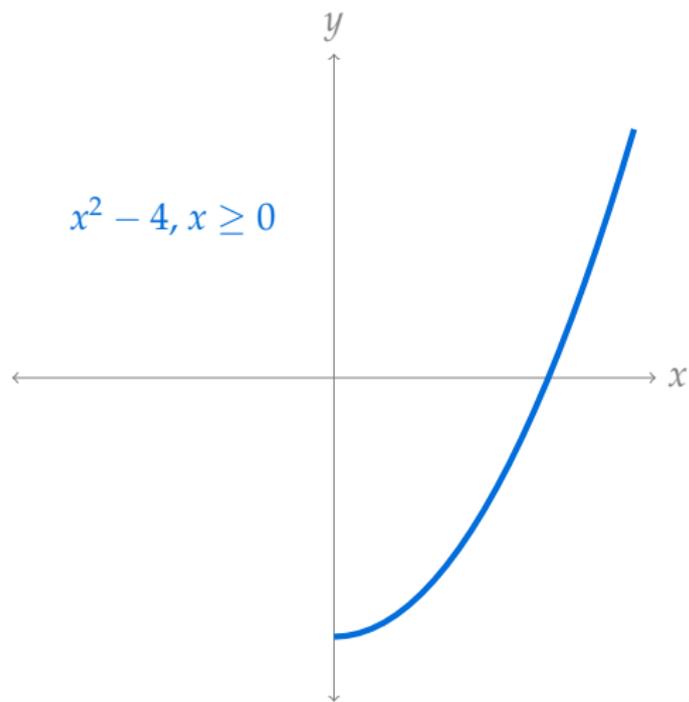
A. invertible

B. not invertible



A. invertible

B. not invertible



A. invertible

B. not invertible

RELATIONSHIP BETWEEN $f(x)$ AND $f^{-1}(x)$

Let f be an invertible function.

What is $f^{-1}(f(x))$?

- A. x
- B. 1
- C. 0
- D. not sure

Invertibility

In order for a function to be invertible, different x values cannot map to the same y value.

We call such a function **one-to-one**, or **injective**.

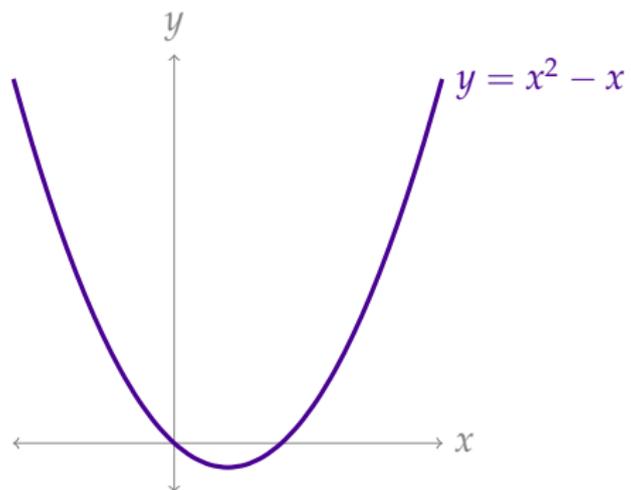
Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

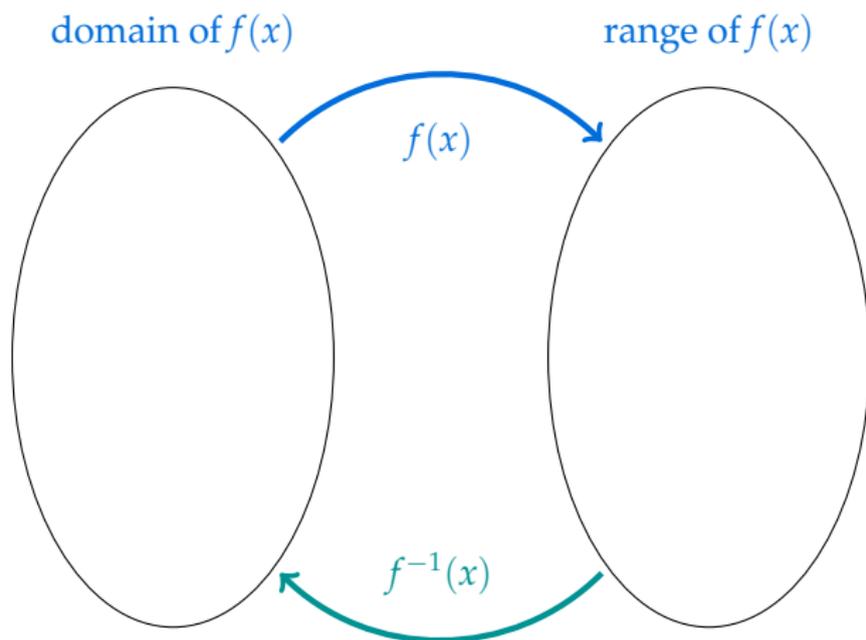
What is $f^{-1}(10)$? (do not simplify)

What is $f^{-1}(x)$?

$$\text{Let } f(x) = x^2 - x.$$

1. Sketch a graph of $f(x)$, and choose a (large) domain over which it is invertible.
2. For the domain you chose, evaluate $f^{-1}(20)$.
3. For the domain you chose, evaluate $f^{-1}(x)$.
4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?





INVERTIBILITY GAME: $f(x) = e^x$

$f^{-1}(x) = \log_e x$

- ▶ I'm thinking of an x . Your clue: $f(x) = e$. What is my x ?
- ▶ I'm thinking of an x . Your clue: $f(x) = 1$. What is my x ?
- ▶ I'm thinking of an x . Your clue: $f(x) = \frac{1}{e}$. What is my x ?
- ▶ I'm thinking of an x . Your clue: $f(x) = e^3$. What is my x ?
- ▶ I'm thinking of an x . Your clue: $f(x) = 0$. What is my x ?

1. Suppose $0 < x < 1$. Then $\log_e(x)$ is...

2. Suppose $-1 < x < 0$. Then $\log_e(x)$ is...

3. Suppose $e < x$. Then $\log_e(x)$ is...

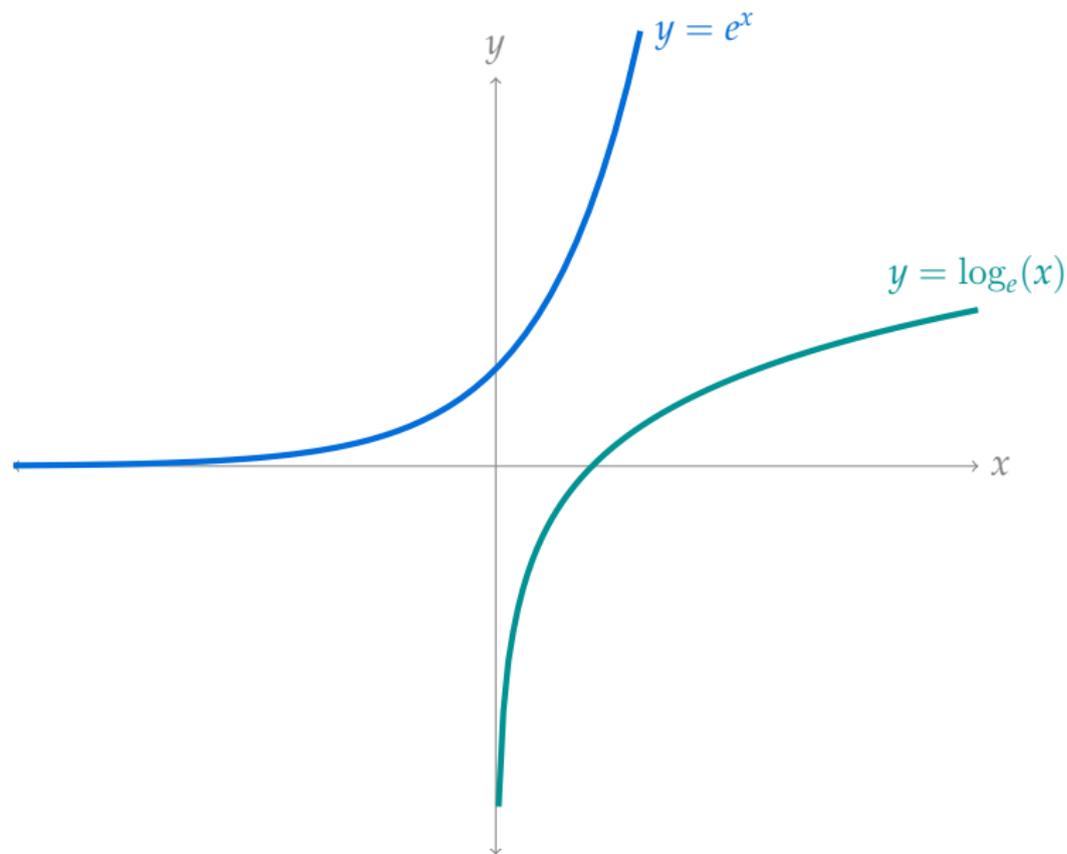
- A. positive
- B. negative
- C. greater than one
- D. less than one
- E. undefined

EXPONENTS AND LOGARITHMS

$$f(x) = e^x$$

$$f^{-1}(x) = \log_e(x) = \ln(x) = \log(x)$$

x	e^x	e fact \leftrightarrow \log_e fact	x	$\log_e(x)$
0	1			
1	e			
-1	$\frac{1}{e}$			
n	e^n			



LOGS OF OTHER BASES: $\log_n(x)$ IS THE INVERSE OF n^x

$$\log_{10} 10^8 =$$

- A. 0
- B. 8
- C. 10
- D. other

$$\log_2 16 =$$

- A. 1
- B. 2
- C. 3
- D. other

Logarithm Rules

Let A and B be positive, and let n be any real number.

$$\log(A \cdot B) = \log(A) + \log(B)$$

$$\text{Proof: } \log(A \cdot B) = \log(e^{\log A} e^{\log B}) = \log(e^{\log A + \log B}) = \log(A) + \log(B)$$

$$\log(A/B) = \log(A) - \log(B)$$

$$\text{Proof: } \log(A/B) = \log\left(\frac{e^{\log A}}{e^{\log B}}\right) = \log(e^{\log A - \log B}) = \log A - \log B$$

$$\log(A^n) = n \log(A)$$

$$\text{Proof: } \log(A^n) = \log\left((e^{\log A})^n\right) = \log(e^{n \log A}) = n \log A$$

Logarithm Rules

Let A and B be positive, and let n be any real number.

$$\log(A \cdot B) = \log(A) + \log(B)$$

$$\log(A/B) = \log(A) - \log(B)$$

$$\log(A^n) = n \log(A)$$

Write as a single logarithm:

$$f(x) = \log\left(\frac{10}{x^2}\right) + 2 \log x + \log(10 + x)$$

BASE CHANGE

$$\begin{aligned}\text{Fact: } \quad & b^{\log_b(a)} = a \\ \Rightarrow & \log(b^{\log_b(a)}) = \log(a) \\ \Rightarrow & \log_b(a) \log(b) = \log(a) \\ \Rightarrow & \log_b(a) = \frac{\log(a)}{\log(b)}\end{aligned}$$

In general, for positive a , b , and c :

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$$

In general, for positive a , b , and c :

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$$

Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\log(17)$?

Suppose your calculator can only compute natural logarithms. What would you enter to calculate $\log_2(57)$?

Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\log(2)$?

Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

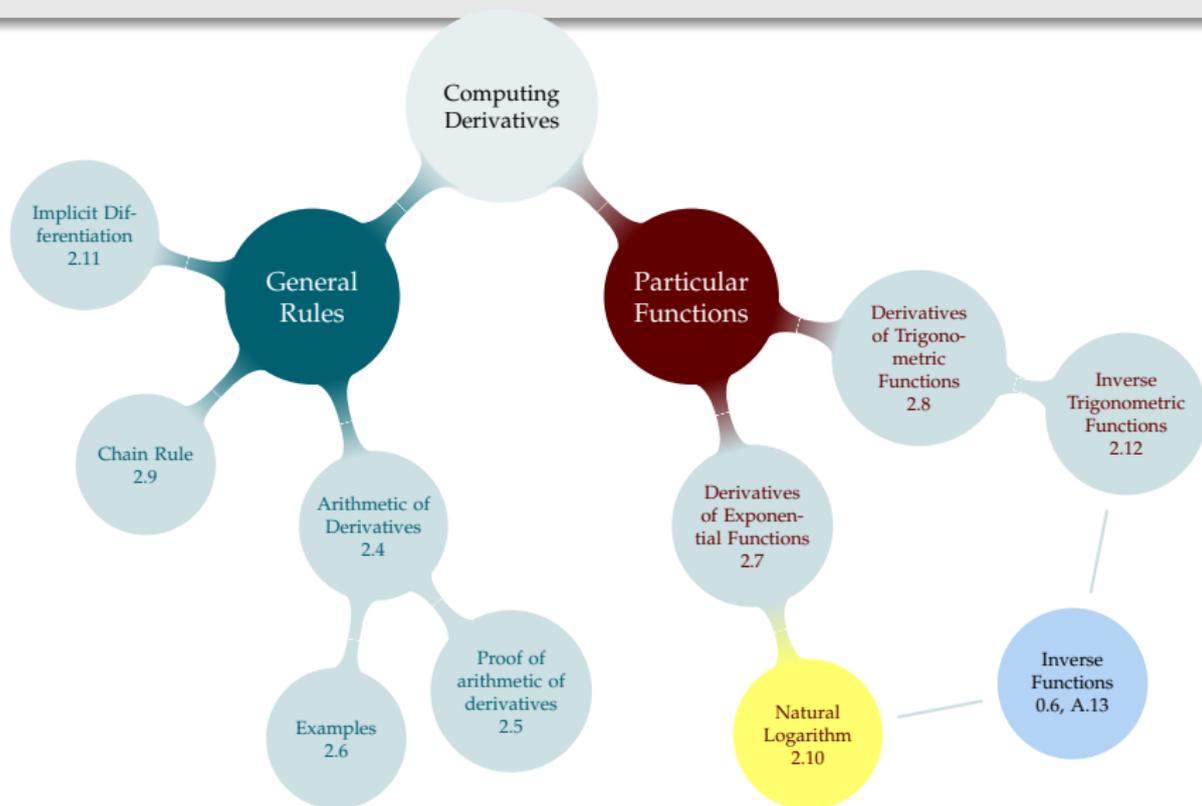
$$10 \log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten **times** louder.

A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

- A. 100 dB
- B. 110 dB
- C. 200 dB
- D. other

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DIFFERENTIATING THE NATURAL LOGARITHM

Calculate $\frac{d}{dx} \{\log_e x\}$.

One Weird Trick:

$$x = e^{\log_e x}$$

$$\frac{d}{dx} \{x\} = \frac{d}{dx} \{e^{\log_e x}\}$$

$$1 = e^{\log_e x} \cdot \frac{d}{dx} \{\log_e x\} = x \cdot \frac{d}{dx} \{\log_e x\}$$

$$\frac{1}{x} = \frac{d}{dx} \{\log_e x\}$$

Derivative of Natural Logarithm

$$\frac{d}{dx} \{\log_e |x|\} = \frac{1}{x} \quad (x \neq 0)$$

Differentiate: $f(x) = \log_e |x^2 + 1|$

Derivatives of Logarithms – Corollary 2.10.6

For $a > 0$:

$$\frac{d}{dx}[\log_a |x|] = \frac{1}{x \log a}$$

In particular:

$$\frac{d}{dx}[\log |x|] = \frac{1}{x}$$

Differentiate: $f(x) = \log_e |\cot x|$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

▶ $\log(f \cdot g) = \log f + \log g$

multiplication turns into addition

▶ $\log\left(\frac{f}{g}\right) = \log f - \log g$

division turns into subtraction

▶ $\log(f^g) = g \log f$

exponentiation turns into multiplication

We can exploit these properties to differentiate!

Logarithmic Differentiation

In general, if $f(x) \neq 0$, $\frac{d}{dx} [\log |f(x)|] = \frac{f'(x)}{f(x)}$.

$$f(x) = \left(\frac{(2x + 5)^4(x^2 + 1)}{x + 3} \right)^5$$

Find $f'(x)$.

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

$$f(x) = \left(\frac{(2x + 5)^4(x^2 + 1)}{x + 3} \right)^5$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

Differentiate:

$$f(x) = x^x$$

LOGARITHMIC DIFFERENTIATION - A FANCY TRICK

Differentiate:

$$f(x) = \left(\frac{(x^{15} - 9x^2)^{10}(x + x^2 + 1)}{(x^7 + 7)(x + 1)(x + 2)(x + 3)} \right)^5$$

$$f(x) = \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x}$$

$$f(x) = (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32})$$

Find $f'(x)$.