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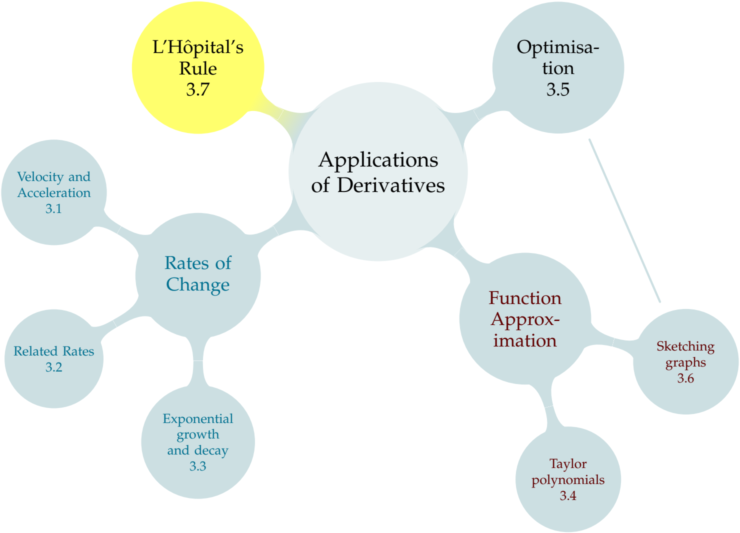

Back to Limits!

$$\lim_{x\to\infty} \frac{x^2}{5} \qquad \lim_{x\to\infty} \frac{5}{x^2} \qquad \lim_{x\to0} \frac{x^2}{5} \qquad \lim_{x\to0} \frac{5}{x^2}$$

Indeterminate Forms – Definition 3.7.1

Suppose $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$. Then the limit

$$\lim_{x\to a} \frac{f(x)}{g(x)}$$

is an indeterminate form of the type $\frac{0}{0}$.

Suppose $\lim_{x\to a} F(x) = \lim_{x\to a} G(x) = \infty$ (or $-\infty$). Then the limit

$$\lim_{x\to a} \frac{F(x)}{G(x)}$$

is an indeterminate form of the type $\frac{\infty}{\infty}$.

When you see an indeterminate form, you need to do more work.

Indeterminate Forms

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}$$

indeterminate form of the type $\frac{0}{0}$

$$\lim_{x \to \infty} \frac{3x^2 - 4x + 2}{8x^2 - 5}$$

indeterminate form of the type $\frac{\infty}{\infty}$

Indeterminate Forms and the Derivative

$$\lim_{x \to 0} \frac{3\sin x - x^4}{x^2 + \cos x - e^x}$$

indeterminate form of the type $\frac{0}{0}$

L'Hôpital's Rule: First Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a, and if $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

L'Hôpital's Rule: Second Part – Theorem 3.7.2

Let f and g be functions such that $\lim_{x\to a} f(x) = \infty = \lim_{x\to a} g(x)$.

If $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{f'(a)}{g'(a)}$.

If f and g are differentiable on an open interval containing a, and if $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$.

This works even for $a = \pm\infty$.

Extremely Important Note:
L'Hôpital's Rule only works on indeterminate forms.

Evaluate:

$$\lim_{x \to 2} \frac{3x \tan(x-2)}{x-2}$$

LITTLE HARDER

$$\lim_{x\to 0} \frac{x^4}{e^x - \cos x - x}$$

indeterminate form of the type $\frac{0}{0}$

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\sqrt{x}}$$

Other Indeterminate Forms

$$\lim_{x\to\infty} e^{-x}\log x \qquad \text{form } 0\cdot\infty$$

VOTE VOTE VOTE

Which of the following can you immediately apply L'Hôpital's rule to?

A. $\dfrac{e^x}{2e^x+1}$

B. $\displaystyle\lim_{x\to 0} \frac{e^x}{2e^x+1}$

C. $\displaystyle\lim_{x\to\infty} \frac{e^x}{2e^x+1}$

D. $\displaystyle\lim_{x\to\infty} e^{-x}(2e^x+1)$

E. $\displaystyle\lim_{x\to 0} \frac{e^x}{x^2}$

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Suppose you want to use L'Hôpital's rule to evaluate $\lim_{x\to a} \frac{f(x)}{g(x)}$, which has the form $\frac{0}{0}$. How does the quotient rule fit into this problem?

A. You should use the quotient rule because the function you are differentiating is a quotient.

B. You will not use the quotient rule because you differentiate the numerator and the denominator separately

C. You may use the quotient rule because perhaps $f(x)$ or $g(x)$ is itself in the form of a quotient

D. You will not use L'Hôpital's rule because $\frac{0}{0}$ is not an appropriate indeterminate form

E. You will not use L'Hôpital's rule because, since the top has limit zero, the whole function has limit 0

More Questions

Which of the following is NOT an indeterminate form?

A. $\frac{\infty}{\infty}$ for example, $\displaystyle\lim_{x\to\infty} \frac{e^x}{x^2}$

B. $\frac{0}{0}$ for example, $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$

C. $\frac{0}{\infty}$ for example, $\displaystyle\lim_{x\to 0^+} \frac{x}{\log x}$

D. $0 \cdot \infty$ for example, $\displaystyle\lim_{x\to\infty} x(\arctan(x) - \pi/2)$

E. all of the above are indeterminate forms

I HAVE SO MANY QUESTIONS

Which of the following is NOT an indeterminate form?

A. 1^∞ for example, $\lim\limits_{x\to\infty} \left(\frac{x+1}{x}\right)^x$

B. 0^∞ for example, $\lim\limits_{x\to\infty} \left(\frac{1}{x}\right)^x$

C. ∞^0 for example, $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$

D. 0^0 for example, $\lim\limits_{x\to 0^+} x^x$

E. all of the above are indeterminate forms

F. none of the above are indeterminate forms

Exponential Indeterminate Forms

$$\lim_{x\to\infty} x^{1/x}$$

Exponential Indeterminate Forms

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$$

Evaluate:

$$\lim_{x\to\infty} \frac{\log x}{\log \sqrt{x}}$$

$$\lim_{x\to\infty} (\log x)^{\sqrt{x}}$$

$$\lim_{x\to 0} \frac{\arcsin x}{x}$$

More Examples

$$\lim_{x\to\infty} \sqrt{2x^2+1} - \sqrt{x^2+x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\sin^2 x}$$

$$\lim_{x\to 0} \sqrt[x^2]{\cos x}$$

Sketch the graph of $f(x) = x \log x$.
Note: when you want to know $\lim_{x \to 0} f(x)$, you'll need to use L'Hôpital.

Evaluate $\lim_{x \to 0^+} (\csc x)^x$