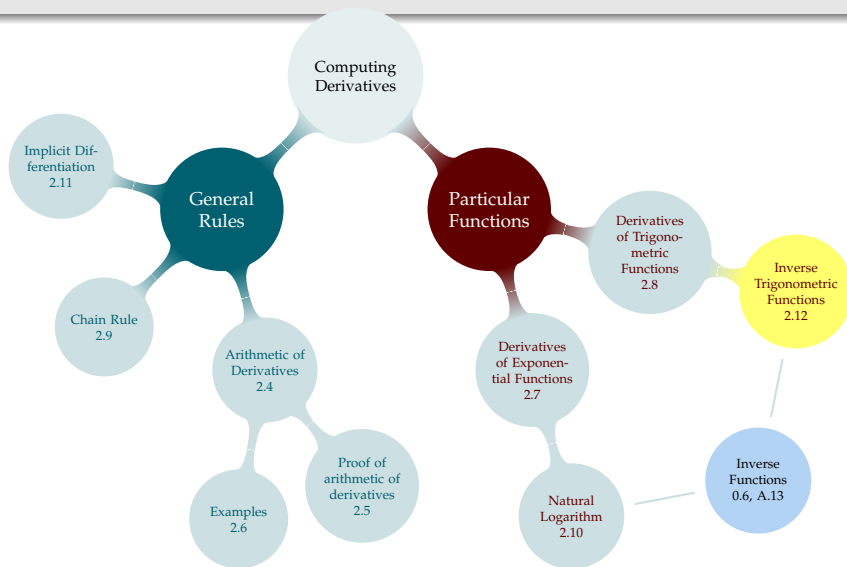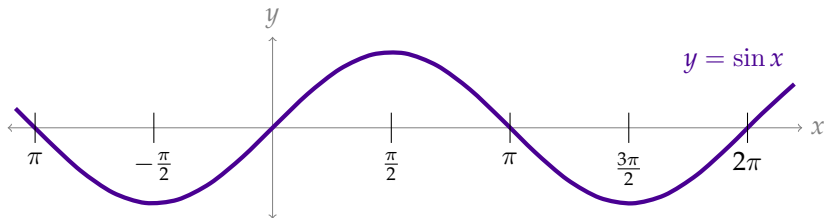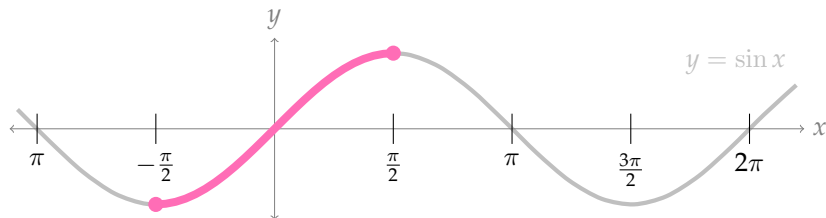# TABLE OF CONTENTS

INVERTIBILITY GAME

I'm thinking of a number x. Your hint: $\sin(x) = 0$. What number am I thinking of?

I'm thinking of a number x, and x is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Your hint: $\sin(x) = 0$. What number am I thinking of?

ARCSINE

$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ is the (unique) number θ such that:

▶ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and

▶ $\sin \theta = x$

ARCSINE

Reference Angles:

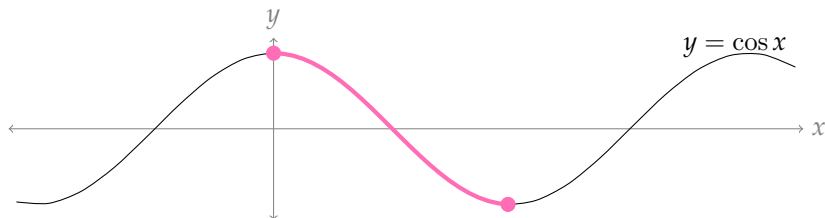| θ	$\sin \theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

▶ $\arcsin(0)$

▶ $\arcsin\left(\frac{1}{\sqrt{2}}\right)$

▶ $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$

▶ $\arcsin\left(\frac{\pi}{2}\right)$

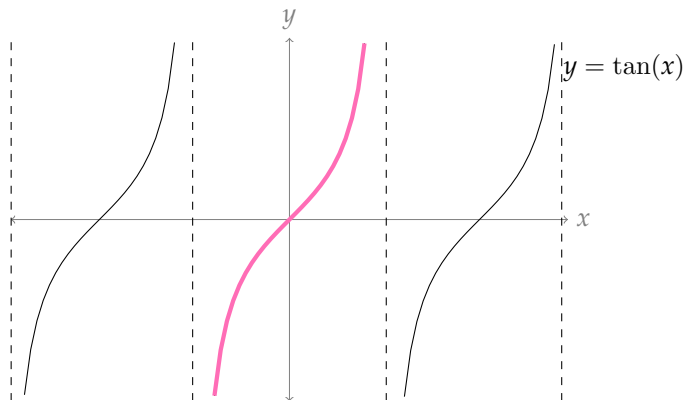▶ $\arcsin\left(\frac{\pi}{4}\right)$

ARCCOSINE

$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$.

$\arccos(x)$ is the (unique) number θ such that:

- ▶ $\cos(\theta) = x$ and
- ▶ $0 \leq \theta \leq \pi$

ARCTANGENT

$\arctan(x) = \theta$ means:
(1) $\tan(\theta) = x$ and
(2) $-\pi/2 < \theta < \pi/2$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\text{arcsec}(x) =$$

ARCSECANT, ARCSINE, AND ARCCOTANGENT

$$\operatorname{arccsc}(x) = \arcsin\left(\tfrac{1}{x}\right)$$

$$\operatorname{arccsc}(x) = y$$
$$\csc y = x$$
$$\frac{1}{\sin y} = x$$
$$\sin y = \frac{1}{x}$$
$$y = \arcsin\left(\tfrac{1}{x}\right)$$
$$\operatorname{arccsc}(x) = \arcsin\left(\tfrac{1}{x}\right)$$

$$\operatorname{arccot}(x) = \arctan\left(\tfrac{1}{x}\right)$$

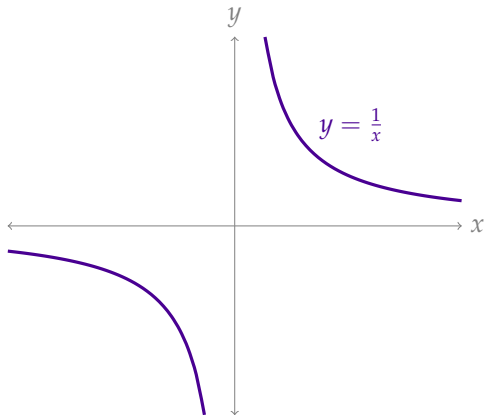$$\operatorname{arccot}(x) = y$$
$$\cot y = x$$
$$\frac{1}{\tan y} = x$$
$$\tan y = \tfrac{1}{x}$$
$$y = \arctan\left(\tfrac{1}{x}\right)$$
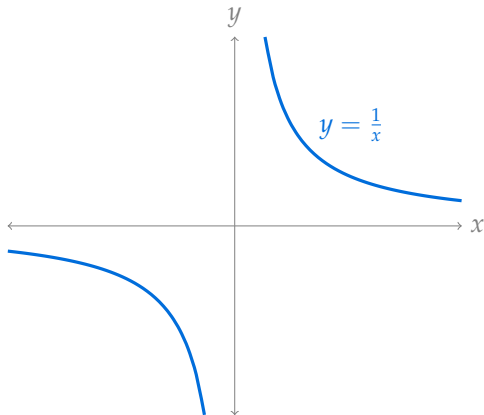$$\operatorname{arccot}(x) = \arctan\left(\tfrac{1}{x}\right)$$

$$\text{arcsec}(x) = \arccos\left(\tfrac{1}{x}\right)$$

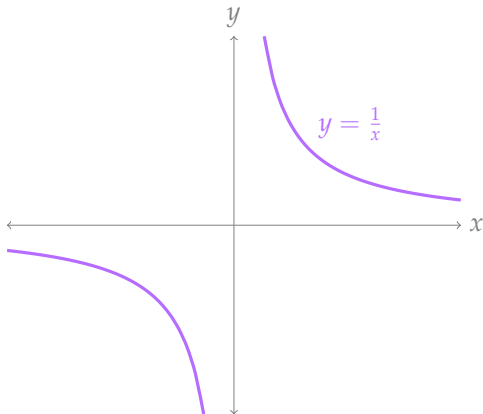The domain of $\arccos(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arcsec}(y)$ is

$y = \frac{1}{x}$

$$\text{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

Domain of $\arcsin(y)$ is $-1 \leq y \leq 1$, so the domain of $\text{arccsc}(x)$ is

$$y = \frac{1}{x}$$

$$\operatorname{arccot}(x) = \arctan\left(\tfrac{1}{x}\right)$$

Domain of $\arctan(x)$ is all real numbers, so the domain of $\operatorname{arccot}(x)$ is

$$y = \arcsin x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y = \arctan x$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y = \arccos x$$

Find $\frac{dy}{dx}$.

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{arccsc}(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(\frac{1}{x}\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\arcsin\left(x^{-1}\right)\right]$$

Derivatives of Inverse Trigonometric Functions – Theorem 2.12.7

Memorize:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x] = \frac{1}{1+x^2}$$

Be able to derive:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$$