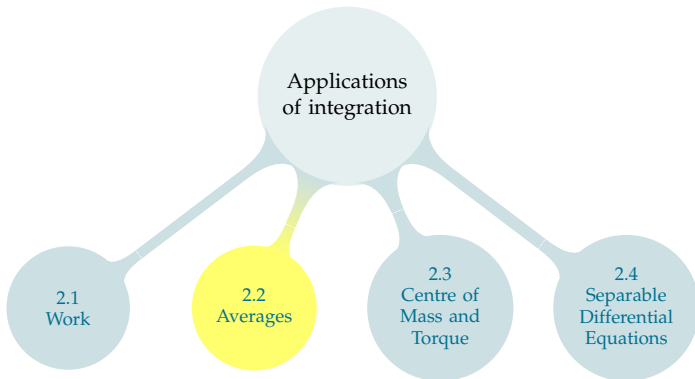
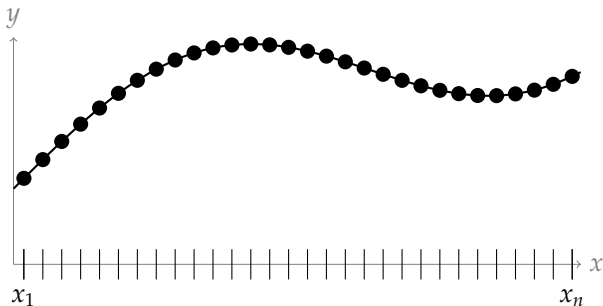


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$$\text{Average} \approx \frac{f(x_1) + \cdots + f(x_n)}{n}$$

$$\begin{aligned} \text{Average} &= \lim_{n \rightarrow \infty} \left[ \frac{1}{n} \sum_{i=1}^n f(x_i) \right] = \lim_{n \rightarrow \infty} \left[ \frac{(b-a)}{(b-a)n} \sum_{i=1}^n f(x_i) \right] \\ &= \lim_{n \rightarrow \infty} \left[ \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x \right] = \frac{1}{b-a} \int_a^b f(x) dx \end{aligned}$$

## Average

Let  $f(x)$  be an integrable function defined on the interval  $a \leq x \leq b$ . The average value of  $f$  on that interval is

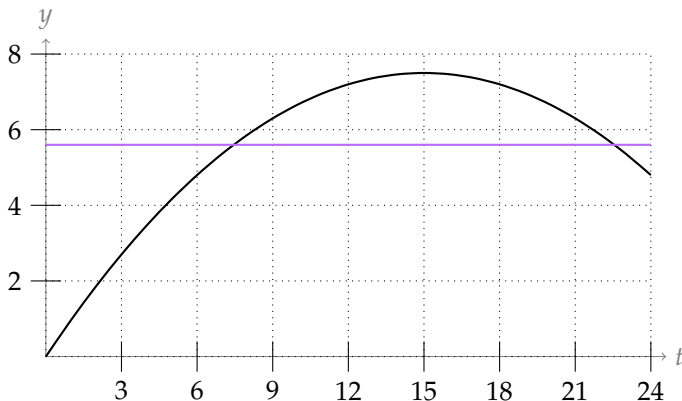
$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

The temperature in a certain city at time  $t$  (measured in hours past midnight) is given by

$$T(t) = t - \frac{t^2}{30}$$

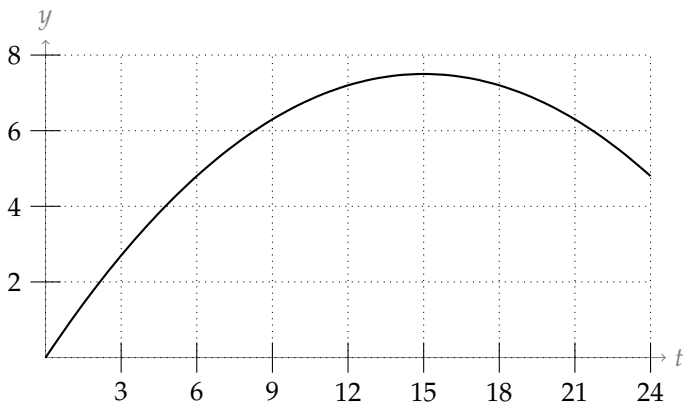
What was the average temperature of one day (from  $t = 0$  to  $t = 24$ )?

Let's check that our answer makes some intuitive sense.

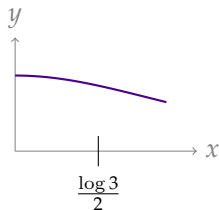


Since the temperature is always between 0 and 8, we expect the average to be between 0 and 8

Let's also recall the motivation for our definition



Find the average value of the function  $f(x) = \frac{e^x}{e^{2x} + 1}$  over the interval  $\left[0, \frac{\log 3}{2}\right]$ .



# AVERAGE VELOCITY

Let  $x(t)$  be the position at time  $t$  of a car moving along the  $x$ -axis. The velocity of the car at time  $t$  is the derivative  $v(t) = x'(t)$ . The average velocity of the car over the time interval  $a \leq t \leq b$  is: