1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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We defined the definite integral using a limit and a sum.

Definition

Let a and b be two real numbers and let $f(x)$ be a function that is defined for all x between a and b. Then we define $\Delta x = \frac{b-a}{N}$ and

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{N\to\infty} \sum_{i=1}^N f(x_{i,N}^*) \cdot \Delta x$$

when the limit exists and when the choice of $x_{i,N}^*$ in the ith interval doesn't matter.

Many of the operations that work nicely with sums and limits will also work nicely with integrals.

ADDING (AND SUBTRACTING) FUNCTIONS
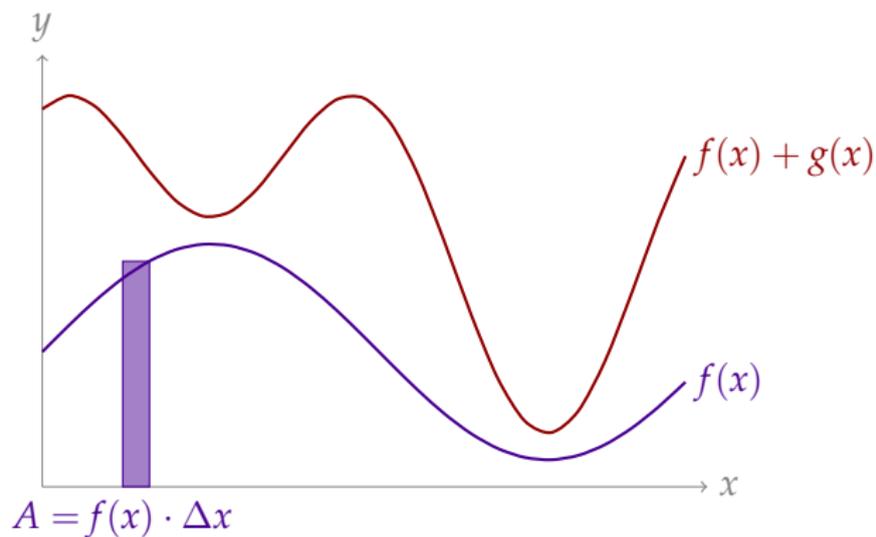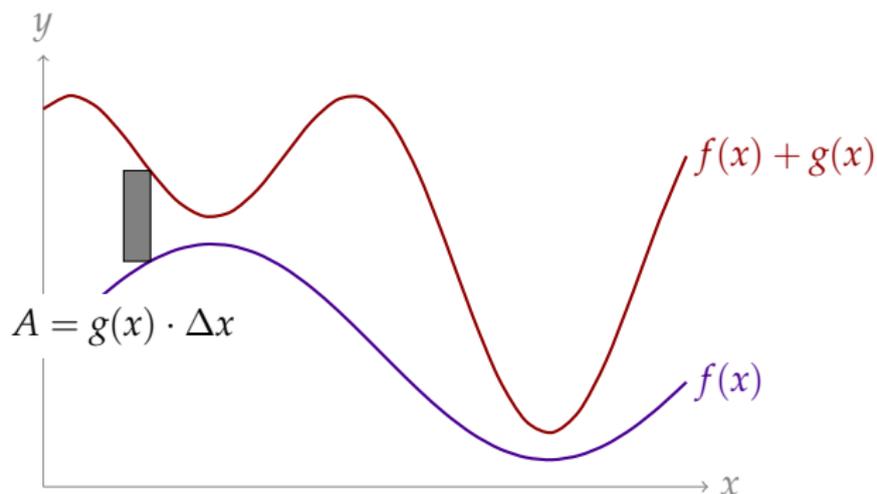
1.2 Basic Properties
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ADDING (AND SUBTRACTING) FUNCTIONS

$$\int_a^b \big(f(x) + g(x)\big)\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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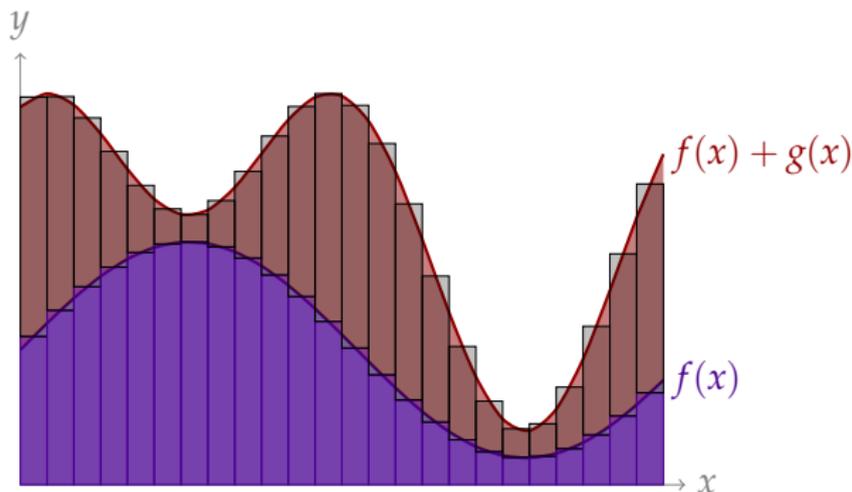
ADDING (AND SUBTRACTING) FUNCTIONS
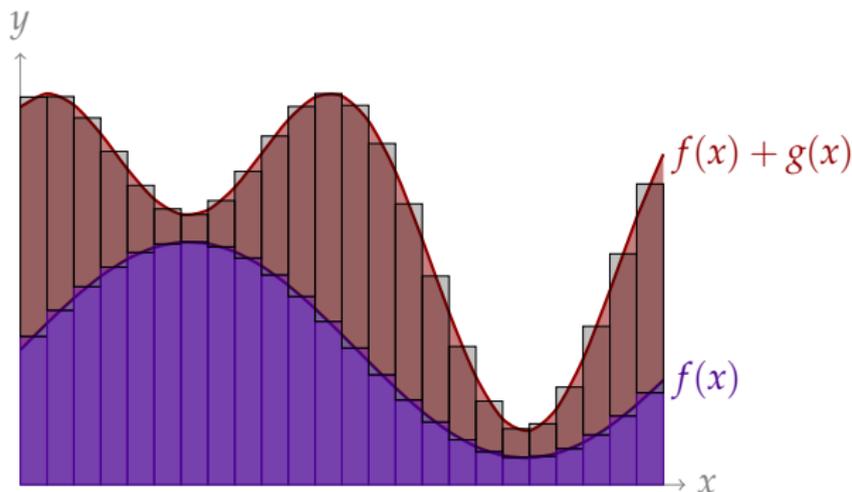
$$\int_a^b \big(f(x) - g(x)\big)\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x - \int_a^b g(x)\,\mathrm{d}x$$

MULTIPLYING A FUNCTION BY A CONSTANT

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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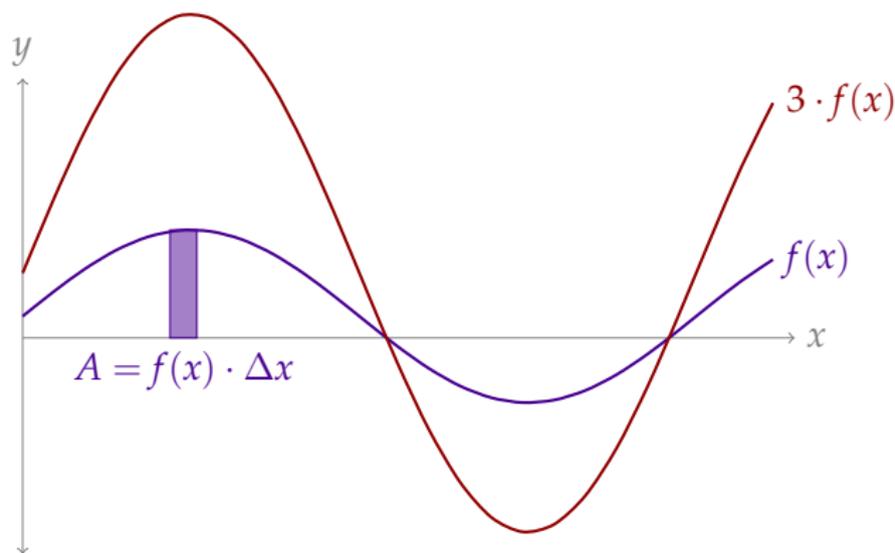
MULTIPLYING A FUNCTION BY A CONSTANT

$$\int_a^b c \cdot f(x)\, \mathrm{d}x = c \int_a^b f(x)\, \mathrm{d}x$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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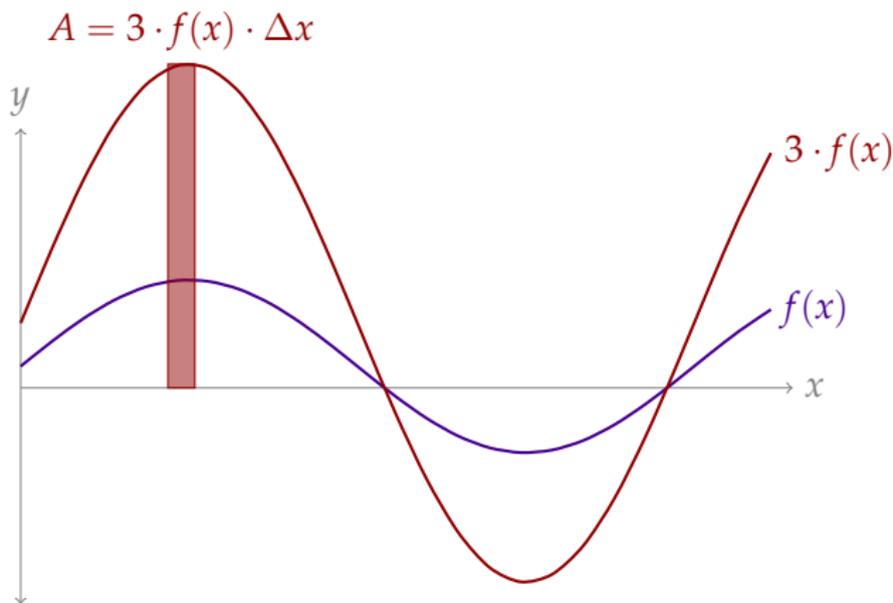
ARITHMETIC OF INTEGRATION

When a, b, and c are real numbers, and the functions $f(x)$ and $g(x)$ are integrable on an interval containing a and b:

(a) $\displaystyle\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(b) $\displaystyle\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

(c) $\displaystyle\int_a^b c \cdot f(x)\,dx = c \int_a^b f(x)\,dx$ when c is constant

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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ARITHMETIC OF INTEGRATION

Suppose $\int_{-1}^{1} f(x)\,\mathrm{d}x = -6$ and $\int_{-1}^{1} g(x)\,\mathrm{d}x = 10$.

$$\int_{-1}^{1} (2f(x) + g(x))\,\mathrm{d}x =$$

$\underset{=}{\mathrm{Q}}$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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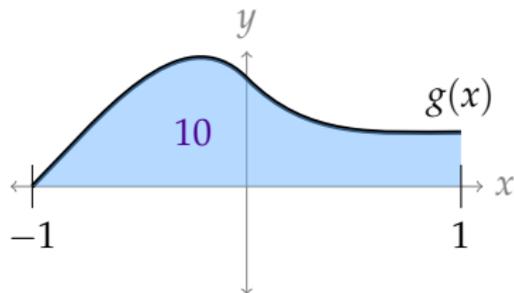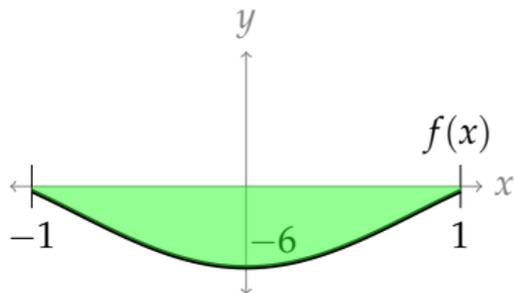
INTERVAL OF INTEGRATION

$$\int_a^a f(x)\, \mathrm{d}x =$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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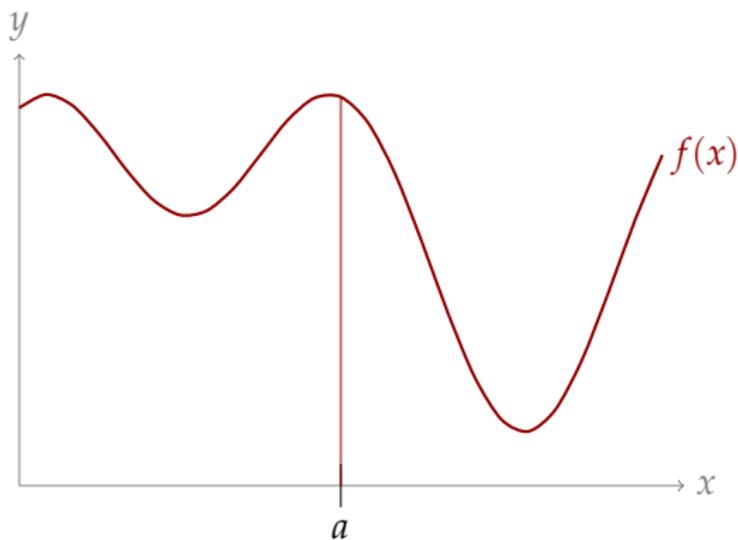
INTERVAL OF INTEGRATION
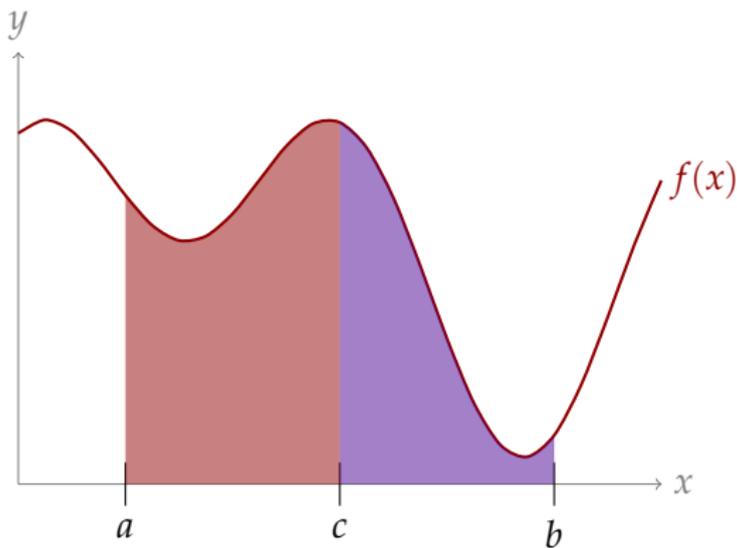
What rule do you think is being illustrated?

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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WHAT HAPPENS IN $\int_a^b f(x)\,\mathrm{d}x$ WHEN $b < a$?
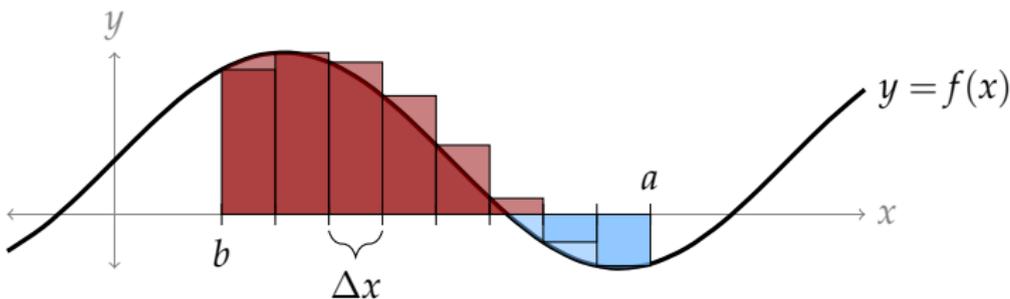
$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i,n}^*) \cdot \frac{b-a}{n}$$

This is the definition of a definite integral *whether or not $a < b$*.

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
○○○○○

PROPERTY OF DEFINITE INTEGRALS

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

As strictly a measure of area, not usually a super useful fact – but helps later when we do arithmetic with integrals.

It's also useful that the definition works without having to worry about which limit of integration (*a* or *b*) is larger.

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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ARITHMETIC FOR DOMAIN OF INTEGRATION

When a, b, and c are constants, and $f(x)$ is integrable over a domain containing all three:

(a) $\displaystyle\int_a^a f(x)\,\mathrm{d}x = 0$
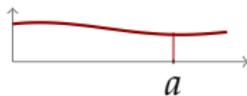
(b) $\displaystyle\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$ $\qquad \Delta x = \dfrac{b-a}{n} = -\dfrac{a-b}{n}$

(c) $\displaystyle\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x$ for constant c

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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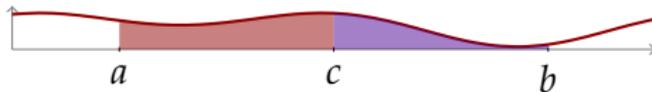
Suppose $\displaystyle\int_{-1}^{0} f(x)\,dx = 1$, $\displaystyle\int_{0}^{1} f(x)\,dx = -3$, and $\displaystyle\int_{-1}^{1} g(x)\,dx = 10$.

$$\int_{-1}^{1} (2f(x) + g(x))\,dx =$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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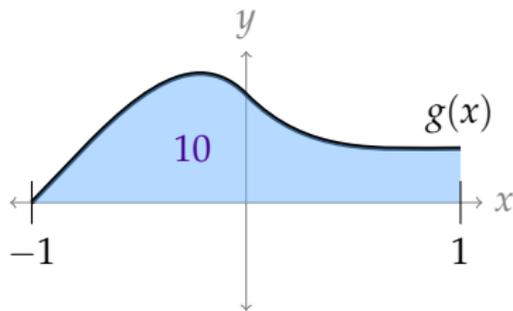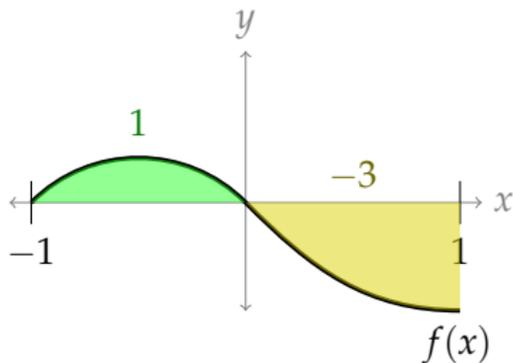
Suppose $\displaystyle\int_{-1}^{0} f(x)\,dx = 1$ and $\displaystyle\int_{0}^{1} f(x)\,dx = -3$.

$$\int_{-1}^{3} f(x)\,dx + \int_{3}^{0} f(x)\,dx =$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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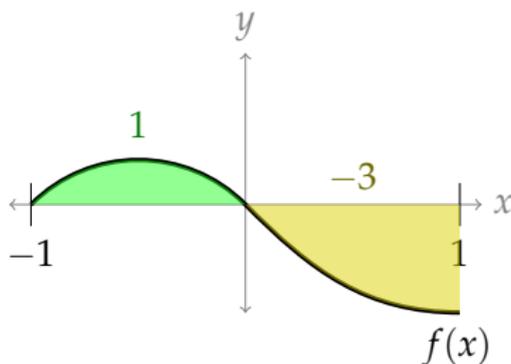
Even and Odd Functions

Let $f(x)$ be a function.

▶ We say $f(x)$ is even when $f(x) = f(-x)$ for all x, and

▶ we say $f(x)$ is odd when $f(x) = -f(-x)$ for all x.

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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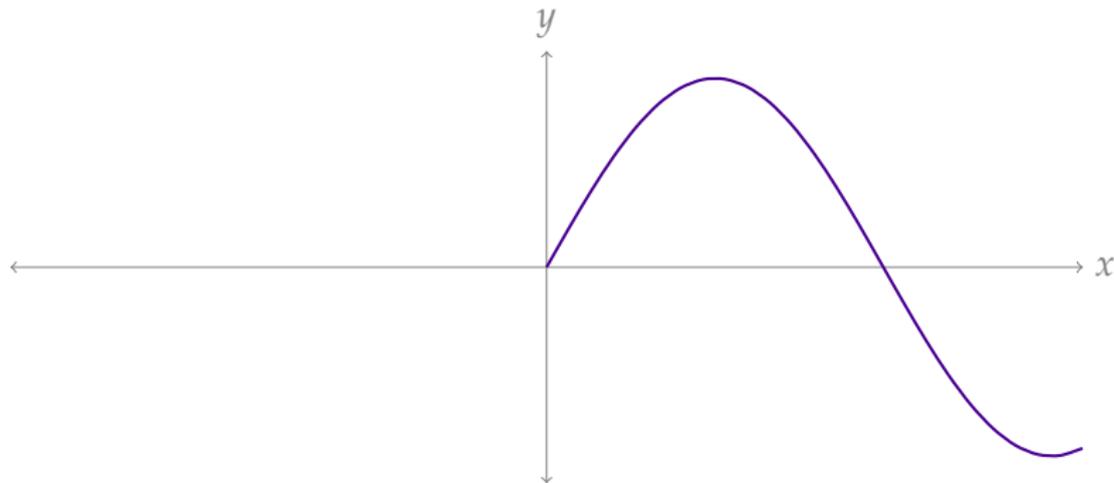
INTEGRALS OF EVEN FUNCTIONS
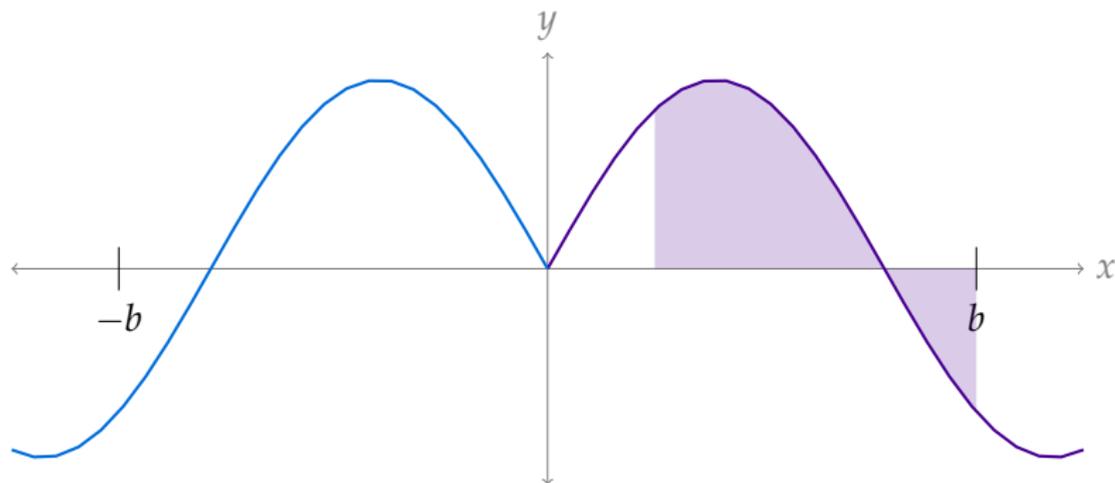
Suppose $f(x)$ is even. Then

$$\int_a^b f(x)\, \mathrm{d}x =$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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INTEGRALS OF EVEN FUNCTIONS
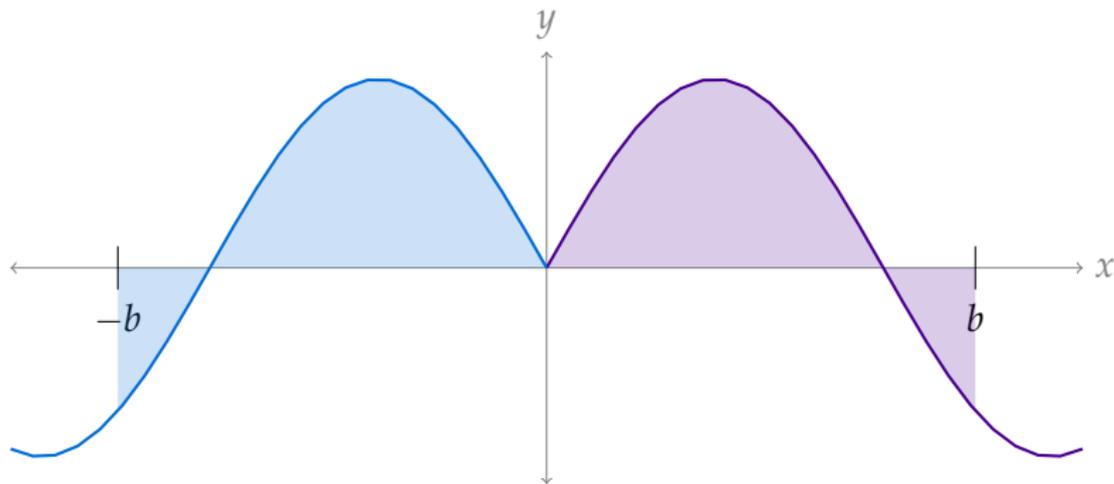
Suppose $f(x)$ is even. Then

$$\int_{-b}^{b} f(x)\,\mathrm{d}x =$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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INTEGRALS OF ODD FUNCTIONS
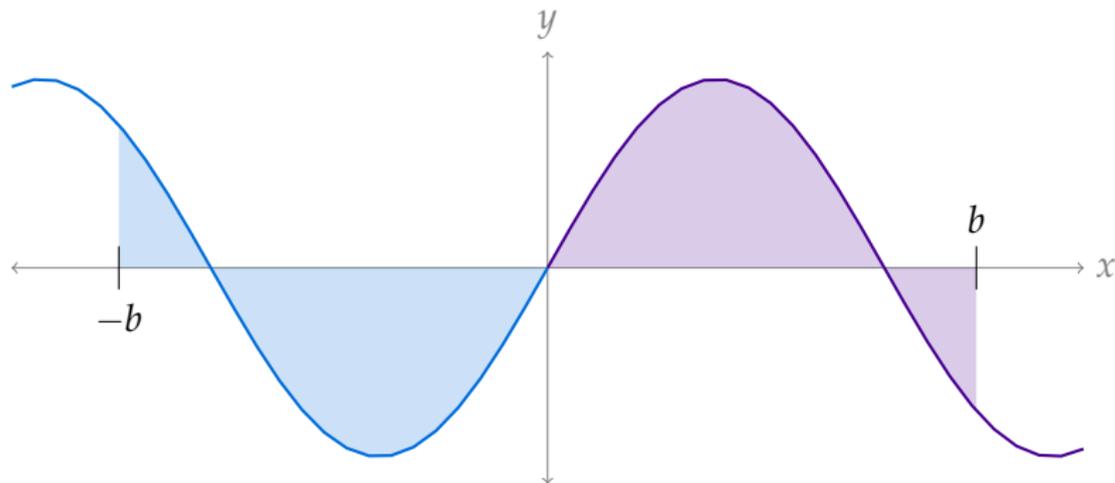
Suppose $f(x)$ is odd. Then

$$\int_{-b}^{b} f(x) \, dx =$$

Theorem 1.2.12 (Even and Odd)

Let $a > 0$.

(a) If $f(x)$ is an even function, then

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 2 \int_{0}^{a} f(x)\,\mathrm{d}x$$

(b) If $f(x)$ is an odd function, then

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0$$

Integral Inequality

Let $a \leq b$ be real numbers and let the functions $f(x)$ and $g(x)$ be integrable on the interval $a \leq x \leq b$.
If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then

$$\int_a^b f(x)\,\mathrm{d}x \leq \int_a^b g(x)\,\mathrm{d}x$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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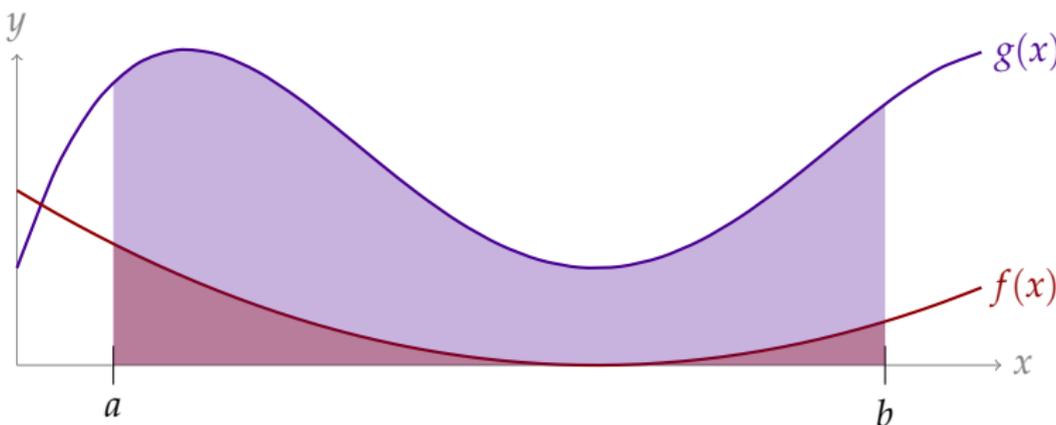
Integral Inequality

Let $a \leq b$ and $m \leq M$ be real numbers and let the function $f(x)$ be integrable on the interval $a \leq x \leq b$.
If $m \leq f(x) \leq M$ for all $a \leq x \leq b$, then

$$m(b - a) \leq \int_a^b f(x)\mathrm{d}x \leq M(b - a)$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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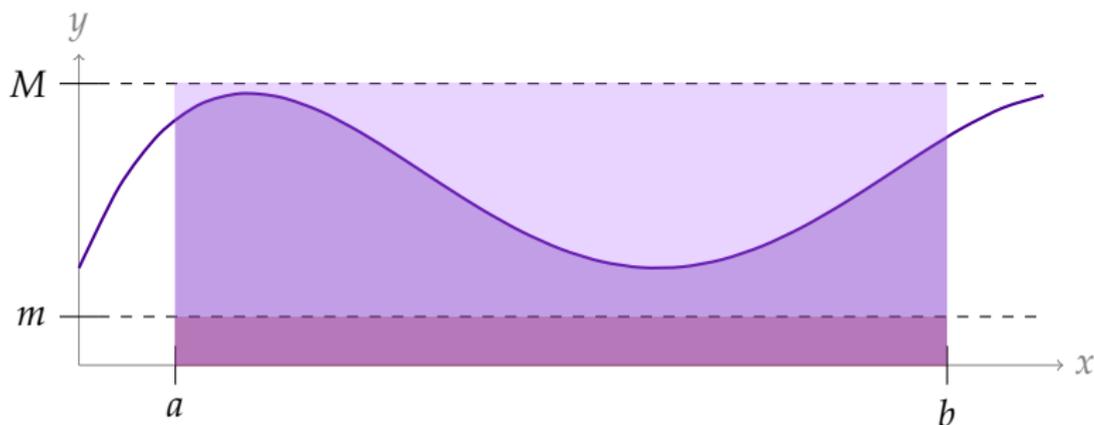
Find a lower bound c and an upper bound d such that

$$c \leq \int_1^5 f(x)\,\mathrm{d}x \leq d$$

1.2 Basic Properties
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1.2.1 Even and Odd Functions
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1.2.2 (Optional) Inequalities
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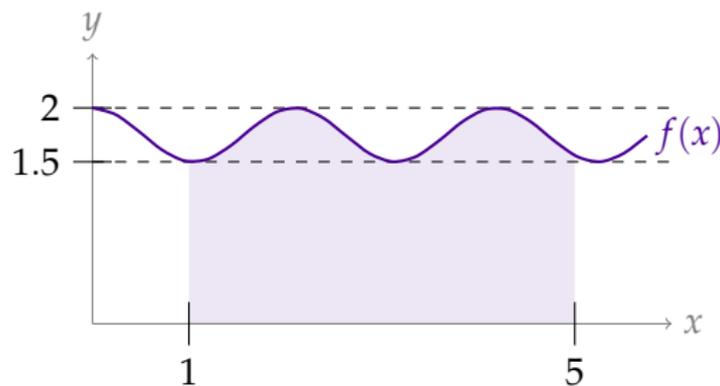
Find a lower bound c and an upper bound d such that $d - c \leq 3$ and

$$c \leq \int_0^6 f(x)\,\mathrm{d}x \leq d$$
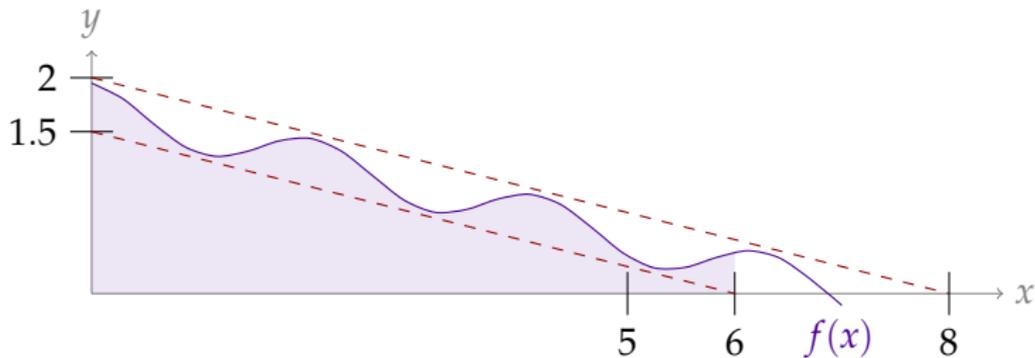
ABSOLUTE VALUES

$f(x) \leq |f(x)|$ for any $f(x)$

$-f(x) \leq |f(x)|$ for any $f(x)$
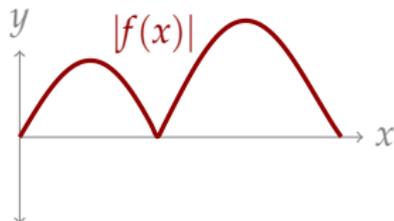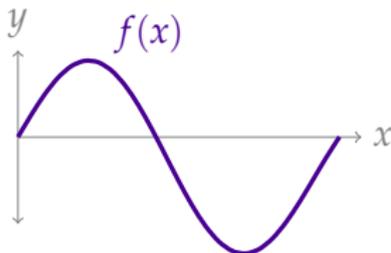
$$\left| \int_a^b f(x)\mathrm{d}x \right| \leq \int_a^b |f(x)|\mathrm{d}x$$

because $\left| \int_a^b f(x)\,\mathrm{d}x \right|$ is either $\int_a^b f(x)\,\mathrm{d}x$ or $-\int_a^b f(x)\,\mathrm{d}x$.