

PLP - 2

TOPIC 2 — LOGICAL STATEMENTS

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LOGICAL STATEMENTS

PROVING THINGS FROM SMALLER THINGS

A big part of mathematics is proving that **statements** are **true**.

The square of an even number is even.

We do this by

- starting from known *facts* — **axioms, lemmas, theorems**
- *combine* those facts using **logic** to build new facts

So we need to introduce and understand

- mathematical sentences that can carry a **truth value**
- the rules for combining those sentences

STATEMENTS

DEFINITION: (A NOT SO FORMAL DEFINITION OF STATEMENTS).

A **statement** is a sentence that is either **true** or **false**.

A statement has exactly one of those **truth values**.

Note:

- A rigorous definition is way too hard for this course.
- Typically use P, Q, R to denote statements

The number $\sqrt{2}$ is not a rational number.

The number 17 is even.

MORE STATEMENTS

Sentences like

The 100th decimal digit of π is 7.

and

Every even integer greater than 2 is the sum of two primes.

are statements since we can determine their truth value

1. $\pi = 3.1415\dots$ — we can just keep computing and check.
2. This is **Goldbach's conjecture** — truth value unknown, but it must be true or false.

NON-STATEMENTS

Sentences like

I am tall

and

This sentence is false

are *not* statements since we cannot decide their truth value

1. Who is “I”? What is “tall”?

2. This cannot be true or false

- If it is true, then it must be false!
- If it is false, then it must be true!

OPEN SENTENCES

Some sentences contain variables

If the integer x is a multiple of 6 then it is even.

The integer x is even.

1. This is a statement since it is always true
2. The truth value depends on the variable x

DEFINITION:

An **open sentence** is a sentence whose truth value depends on the variable(s) it contains.

Typically denote it by $P(x)$ and similar.