# PLP - 2 TOPIC 2 — LOGICAL STATEMENTS

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# LOGICAL STATEMENTS

# **PROVING THINGS FROM SMALLER THINGS**

### A big part of mathematics is proving that statements are true.

The square of an even number is even.

We do this by

- starting from known *facts* **axioms**, **lemmas**, **theorems**
- *combine* those facts using **logic** to build new facts

So we need to introduce and understand

- mathematical sentences that can carry a truth value
- the rules for combining those sentences

# **STATEMENTS**

# **DEFINITION: (A NOT SO FORMAL DEFINITION OF STATEMENTS).**

A **statement** is a sentence that is either **true** or **false**.

A statement has exactly one of those truth values.

Note:

- A rigorous definition is way too hard for this course.
- Typically use P, Q, R to denote statements

The number  $\sqrt{2}$  is not a rational number.

The number 17 is even.

# **MORE STATEMENTS**

#### Sentences like

The 100th decimal digit of  $\pi$  is 7.

#### and

*Every even integer greater than 2 is the sum of two primes.* 

are statements since we can determine their truth value

1.  $\pi = 3.1415...$  — we can just keep computing and check.

2. This is **Goldbach's conjecture** — truth value unknown, but it must be true or false.

# **NON-STATEMENTS**

#### Sentences like

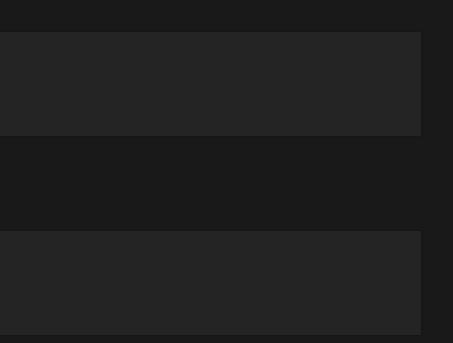
I am tall

and

#### This sentence is false

are *not* statements since we cannot decide their truth value

- 1. Who is "I"? What is "tall"?
- 2. This cannot be true or false
  - If it is true, then it must be false!
  - If it s false, then it must be true!



### **OPEN SENTENCES**

#### Some sentences contain variables

If the integer x is a multiple of 6 then it is even.

The integer x is even.

This is a statement since it is always true
The truth value depends on the variable x

# **DEFINITION:**

An open sentence is a sentence whose truth value depends on the variable(s) it contains.

Typically denote it by P(x) and similar.