PLP - 3

TOPIC 3 — AND, OR & NOT

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AND, OR & NOT

NEGATION

Given P we can form a new statement with the opposite truth value.

> *It is not the case that P*

DEFINITION:

The **negation** of a statement P is denoted $\sim P$.

- When P is *true*, the negation $\sim P$ is *false*.
- When P is *false*, the negation $\sim P$ is *true*.

The negation is also denoted $!P$ and $\neg P$.

EXAMPLES AND A TABLE

- The negation of “It is tuesday” is “It is *not* Tuesday”
- The negation of “$4 \in A$” is “$4 \notin A$”
- The negation of “4 is even” is “4 is not even” or better “4 is odd”.

We can summarise what negation does to truth values via a **truth table**

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

Note

- the **double negation** of P has the same truth value as P
- the **law of the excluded middle**: exactly one of P or $(\sim P)$ is true.

CONJUNCTION, AND, DISJUNCTION, & OR

We combine statements using **and** & **or** to make new statements.

The words “and”, “or” have precise mathematical meanings

DEFINITION:

Let P and Q be statements.

- The **disjunction** of P and Q is “P **or** Q” and is denoted $P \vee Q$.

  $P \vee Q$ is true when at least one of P, Q is true, else false.

- The **conjunction** of P and Q is “P **and** Q” and is denoted $P \wedge Q$.

  $P \wedge Q$ is true when both P, Q are true, else false.

Note: *colloquial* use of “or” is often different from this *mathematical* “or”

EXAMPLES AND TABLES

Let P be "8 is even" and let Q be "15 is prime", then

- $P \vee Q$ is "8 is even or 15 is prime"
- $P \wedge Q$ is "8 is even and 15 is prime"

The first is true since P is true, the second is false since Q is false.

A truth table helps summarise:

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

INCLUSIVE AND EXCLUSIVE

Mathematical “or”or is **inclusive** — $P \vee Q$ is true when *at least one* statement is true.

Colloquial “or” is often **exclusive** — $P \text{ xor } Q$ is true when *exactly one* statement is true.

> *Would you like chicken or beef for dinner?*

P	Q	$P \vee Q$	$P \text{ xor } Q$
T	T	T	**F**
T	F	T	T
F	T	T	T
F	F	F	F

For exclusive-or write “Exactly one of P or Q” or “P or Q but not both”.