PLP - 6

TOPIC 6 —CONVERSE, CONTRAPOSITIVE AND BICONDITIONAL

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CONVERSE, CONTRAPOSITIVE AND BICONDITIONAL

CONVERSE AND CONTRAPOSITIVE

DEFINITION: (CONVERSE & CONTRAPOSITIVE).

Given the implication $P \implies Q$ we can form

- the **converse**: $Q \implies P$, and
- the **contrapositive**: $(\sim Q) \implies (\sim P)$

P	Q	$P \implies Q$	$Q \implies P$	$(\sim Q) \implies (\sim P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

DEATH AND SHAKESPEARE

> *If he is Shakespeare then he is dead*

The **contrapositive** is the same as the original

> *If he is not dead then he is not Shakespeare*

The **converse** is *not* the same as the original

> *If he is dead then he is Shakespeare*

Frequent source of bad logic — be careful!

CONVERSE

The converse can lead to interesting results. Once we prove

If n is even then n^2 is even

It makes sense to ask if the converse is true

If n^2 is even then n is even

Much interesting mathematics comes from considering whether the converse is also true.

BICONDITIONAL

We use the **biconditional** to express $(P \implies Q)$ and $(Q \implies P)$ both true

DEFINITION: (BICONDITIONAL).

The **biconditional** is the statement "P if and only if Q" and is denoted "$P \iff Q$" and "P iff Q".

The biconditional is true when P and Q have the same truth value and otherwise false.

P	Q	$P \implies Q$	$Q \implies P$	$P \iff Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T