PLP - 8

TOPIC 8 —A FIRST PROOF

Demirbaş & Rechnitzer

A FIRST PROOF

A FIRST RESULT

Our very first result will be

> **PROPOSITION:**
>
> Let n be an integer. If n is even then n^2 is even

We want to show this implication is always true.

- When *hypothesis is false* (n is not even) then implication is true — no work required!
- So *assume hypothesis is true* — n is an even number.
- $\dots$
- So n^2 must be an even number

Clearly we need to understand **even** — we need the definition.

Important — memorise definitions

CONTINUING

PROPOSITION:

Let n be an integer. If n is even then n^2 is even

- Start by *assuming the hypothesis is true:* so we assume that n is even.
- *By the definition of even* we know that $n = 2k$ for some integer k.
- But then, $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$.
- Since $k \in \mathbb{Z}$ we know *by an axiom* that $2k^2 \in \mathbb{Z}$.
- So *by the definition of even* we know that n^2 is even

WHAT JUST HAPPENED?

What have we done? We showed all these implications

- (n is even) $\implies$ ($n = 2k$ for some integer k)
- ($n = 2k$ for some integer k) $\implies$ ($n^2 = 4k^2$)
- ($n^2 = 4k^2$) $\implies$ (n^2 is two times an integer)
- (n^2 is two times an integer) $\implies$ (n^2 is even)

So when we assume n is even, we can use **modus ponens** to see that

- ($n = 2k$) is true
- ($n^2 = 4k^2$) is true
- (n^2 is two times an integer) is true
- (n^2 is even) is true

So when the **hypothesis** is true, **the conclusion** must be true, and so the **implication** is true!

Our first proof! — nearly.

CLEANING UP

When "doing" proofs we nearly always separate **scratch work** from the **proof**.

Scratch work

All our draft work — the **reader** doesn't need to see this.

The proof

The cleaned up work, neatly formatted, so easy for the **reader** to follow

PROOF.

- Assume that n is an even number.
- Hence we know that $n = 2k$ for some $k \in \mathbb{Z}$.
- It follows that $n^2 = 4k^2 = 2(2k^2)$
- Since $2k^2$ is an integer, it follows that n^2 is even