PLP - 11

TOPIC 11 —CONTRAPOSITIVE PROOF

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CONTRAPOSITIVE PROOF

TRY THE CONTRAPOSITIVE

PROPOSITION:

Let $n \in \mathbb{Z}$. If n^2 is even then n is even.

Scratch work

- *Assume hypothesis is true* so n^2 is even
- Hence $n^2 = 2k$ for some integer k
- So $n = \sqrt{2k}$ and so …

Not sure where to go? Try the **contrapositive**

TRY THE CONTRAPOSITIVE 2

Let $n \in \mathbb{Z}$. If n^2 is even then n is even.

Scratch work

- Form the contrapositive: If n is not even then n^2 is not even
- Since $n \in \mathbb{Z}$: If n is odd then n^2 is odd
- Now we know what to do — just tell the **reader** that you are proving the contrapositive.

PROOF.

We prove the contrapositive: if n is odd then n^2 is odd.

- Assume that n is odd.
- Hence $n = 2\ell + 1$ and so $n^2 = 4\ell^2 + 4\ell + 1 = 2(2\ell^2 + 2\ell) + 1$.
- Since $2\ell^2 + 2\ell \in \mathbb{Z}$, it follows that n^2 is odd.

Since the contrapositive it true, the original statement is true.

ANOTHER EXAMPLE

PROPOSITION:

Let $n \in \mathbb{Z}$. If $3n+7$ is odd then n is even.

Scratch work

- Assume $3n+7$ is odd, so $3n+7 = 2\ell+1$
- Then $3n = 2\ell - 6$ and $n = \frac{2\ell-6}{3}$ which is… **stuck**
- Start again with *contrapositive*: If n is odd then $3n+7$ is even.
- Then $n = 2k+1$ so $3n+7 = 6k+3+7 = 2(3k+5)$ which is even.

Write it up nicely.

WRITE UP THE PROOF

PROOF.

We prove the contrapositive. Assume that n is odd, so $n = 2k + 1$ for some $k \in \mathbb{Z}$. Then $3n + 7 = 2(3k + 5)$ and since $3k + 5 \in \mathbb{Z}$ it follows that $3n + 7$ is even.

Since the contrapositive is true, the result holds.

Usually more than 1 way to prove things. A direct proof:

PROOF.

Let $3n + 7$ be odd, so $3n + 7 = 2k + 1$ for some $k \in \mathbb{Z}$. But then

$$3n = 2k - 6 \qquad \text{and so} \qquad n = 2k - 6 - 2n = 2(k - 3 - n)$$

Now since $k - 3 - n \in \mathbb{Z}$ it follows that n is even.