PLP - 20

TOPIC 20—MORE INDUCTION

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MORE EXAMPLES

AN INEQUALITY

PROPOSITION:

Let $x > -1$, then for all $n \in \mathbb{N}$, $(1+x)^n \geq 1 + nx$

Scratch work

- When $n = 1$ we have $(1+x) = 1 + x$, so all good
- Assume that $(1+x)^k \geq (1+kx)$, so

$$\begin{aligned}(1+x)^{k+1} &= (1+x)\cdot(1+x)^k \\ &\geq (1+x)(1+kx) = 1+(k+1)x+kx^2 \\ &\geq 1+(k+1)x \qquad \text{since } x^2 \geq 0\end{aligned}$$

Where did we use $x > -1$?

WRITE IT UP NICELY

PROOF.

We proceed by induction. Assume that $x > -1$.

- Base case: When $n = 1$ we have $(1 + x) = (1 + x)$, as required
- Inductive step: Assume that the result holds for $n = k$, so $(1+x)^k \geq (1 + kx)$. Then

$$
\begin{aligned}
(1+x)^{k+1} &\geq (1+x)(1+kx) && \text{since } 1 + x > 0 \\
&= 1 + (k+1)x + kx^2 \\
&\geq 1 + (k+1)x && \text{since } kx^2 \geq 0
\end{aligned}
$$

and so the result holds for $n = k + 1$

ANOTHER EXAMPLE

PROPOSITION:

For all $n \in \mathbb{N}$, $1 + 3 + \cdots + (2n - 1) = n^2$.

Scratch work

- Base case: When $n = 1$ we have $(2 - 1) = 1^2$.
- Inductive step: Assume $1 + 3 + \cdots + (2k - 1) = k^2$ then

$$\begin{aligned} 1 + 3 + \cdots + (2k - 1) + (2k + 1) &= k^2 + (2k + 1) \\ &= (k + 1)^2 \end{aligned}$$

as required.

Warning do not think "*add the next term*". It is "$P(k) \implies P(k + 1)$"

WRITE IT UP

PROOF.

We prove the result by induction.

- Base case: when $n = 1$, we have $(2-1) = 1^2$, as required.
- Inductive step: assume that $1 + 3 + \cdots + (2k-1) = k^2$, but then

$$1 + 3 + \cdots + (2k-1) + (2k+1) = k^2 + 2k + 1 = (k+1)^2$$

  Hence the inductive step holds.

So by induction the result holds for all $n \in \mathbb{N}$.

Warning inductive step is not "*add the next term*". It is "$P(k) \implies P(k+1)$"