PLP - 22

TOPIC 22—SUBSETS AND POWER SETS

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SUBSETS

DOING MORE WITH SETS

Is the set A contained in the set B?

DEFINITION: SUBSET.

Let A, B be sets

- We say that A is a **subset** of B when every element of A is also an element of B.
- We denote this $A \subseteq B$ and also call B a **superset** of A. We can also write $B \supseteq A$.
- A is a **proper subset** of B when $A \subseteq B$, but B contains at least one element that is not in A.
- Finally, two A and B are equal when they are subsets of each other. That is

$$A = B \iff ((A \subseteq B) \wedge (B \subseteq A))$$

NOTES AND EXAMPLES

Note that

- For all sets A, $\varnothing \subseteq A$ and $A \subseteq A$
- $A \subseteq B \quad \equiv \quad \forall a \in A, a \in B \quad \equiv \quad (a \in A) \implies (a \in B)$
- $A \nsubseteq B \quad \equiv \quad \exists a \in A \ s.t. \ a \notin B$

Some examples

- $\{1, 2, 7\} \nsubseteq \{1, 2, 3, 4, 5\}$
- $\{2n \ : \ n \in \mathbb{Z}\} \subseteq \mathbb{Z}$
- The subsets of $\{0, 1\}$ are $\varnothing$, $\{0\}$, $\{1\}$, $\{0, 1\}$

THE SET OF ALL SUBSETS

DEFINITION:

Let A be a set. The **power set** of A, denoted $\mathcal{P}(A)$, is the set of all subsets of A.

$$\begin{aligned}
\mathcal{P}(\varnothing) &= \{\varnothing\} \\
\mathcal{P}(\{1\}) &= \{\varnothing, \{1\}\} \\
\mathcal{P}(\{0,1\}) &= \{\varnothing, \{0\}, \{1\}, \{0,1\}\} \\
\mathcal{P}(\{0,1,2\}) &= \{\varnothing, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}
\end{aligned}$$

Not hard to prove that if $|A| = n$ then $|\mathcal{P}(A)| = 2^n$.

Near end of course we'll prove a *very interesting* result for infinite sets A and their power sets.