PLP - 23

TOPIC 23—SET OPERATIONS

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SET OPERATIONS

UNION AND INTERSECTION

DEFINITION:

Let A, B be sets. The **union** of A and B is

$$A \cup B = \{x \ : \ x \in A \text{ or } x \in B\}$$

DEFINITION:

The **intersection** of sets A and B, is

$$A \cap B = \{x \ : \ x \in A \text{ and } x \in B\}$$

If the intersection $A \cap B = \varnothing$, then we say that A and B are **disjoint**.

NOTES AND EXAMPLES

- Please use correct notation

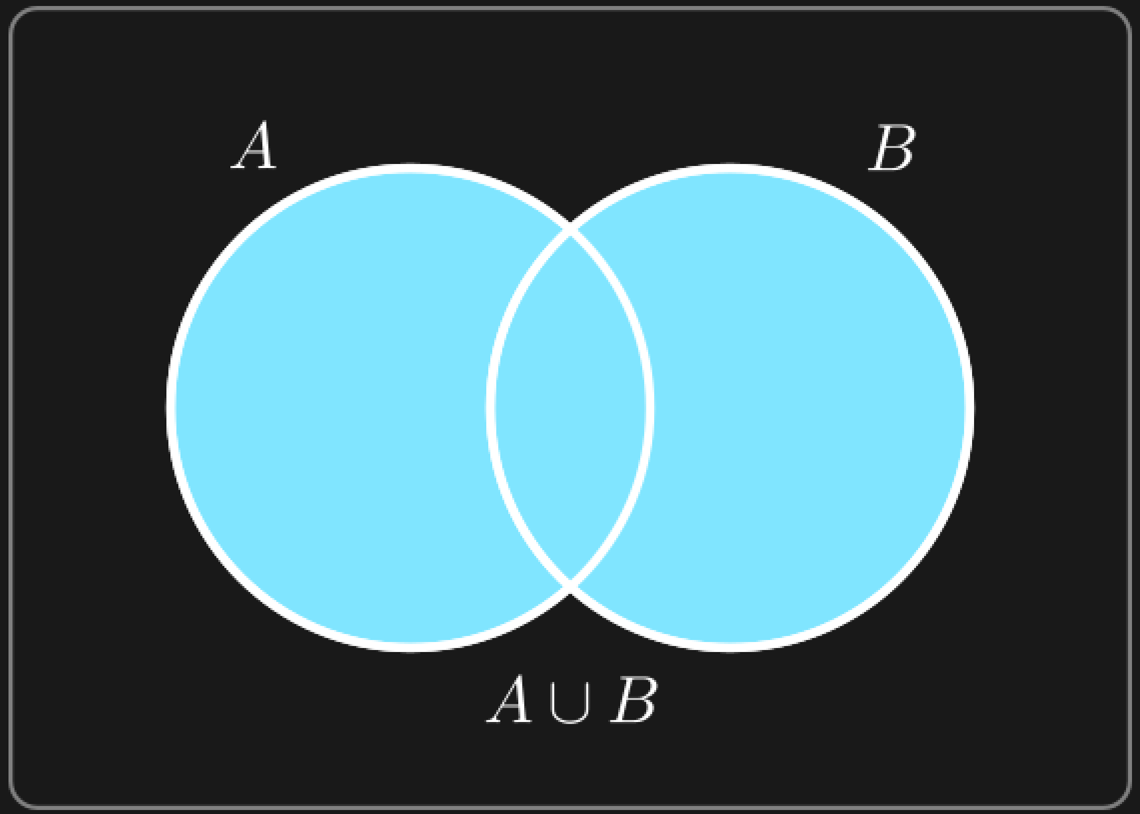
$$A \cup B = \text{ unions of sets} \qquad A \cap B = \text{ intersection of sets}$$
$$P \vee Q = \text{ disjunction of statements} \qquad P \wedge Q = \text{ conjunction of statements}$$

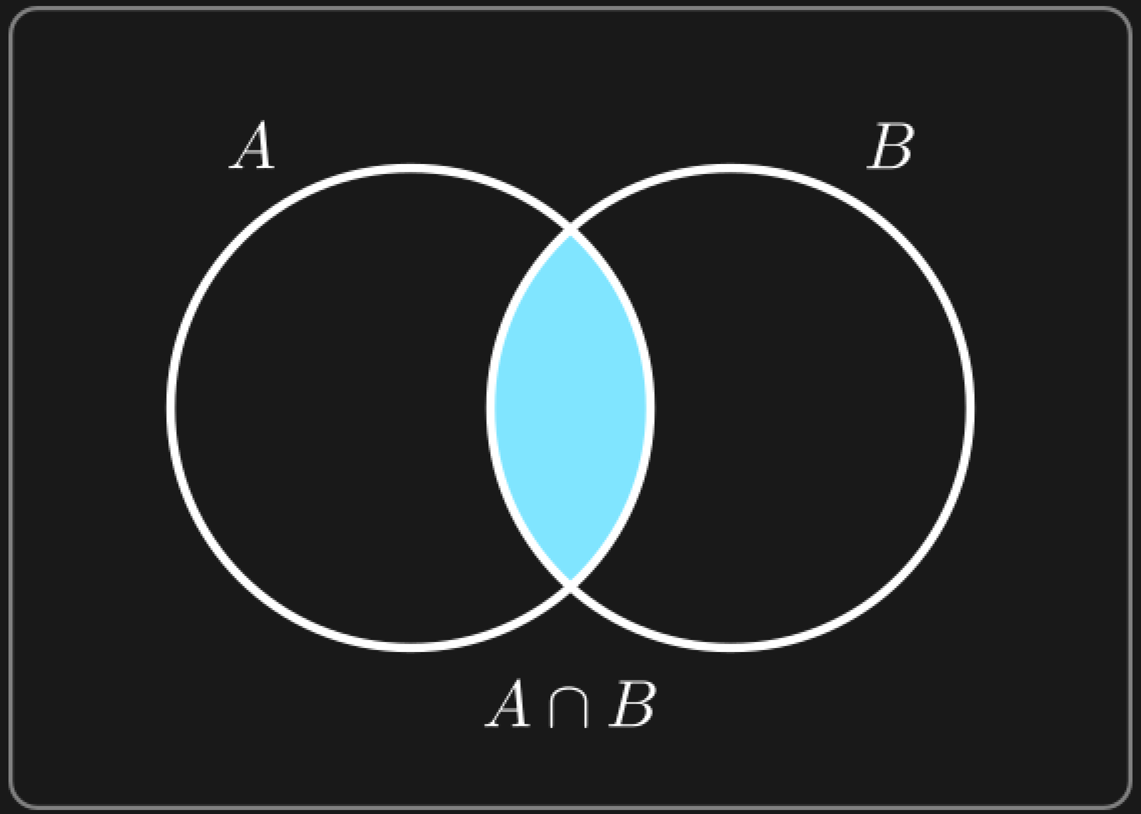
- There are *parallels* between set operations and logical operations but they are *not* the same
- Let $A = \{1, 2, 3, 4\}$, $B = \{p \;:\; p \text{ is prime}\}$ and $C = \{4, 5, 6, 7\}$

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7\} \qquad A \cap B = \{2, 3\} \qquad B \cap C = \{5, 7\}$$

VISUALISING THINGS

We can picture union and intersection using **Venn diagrams**



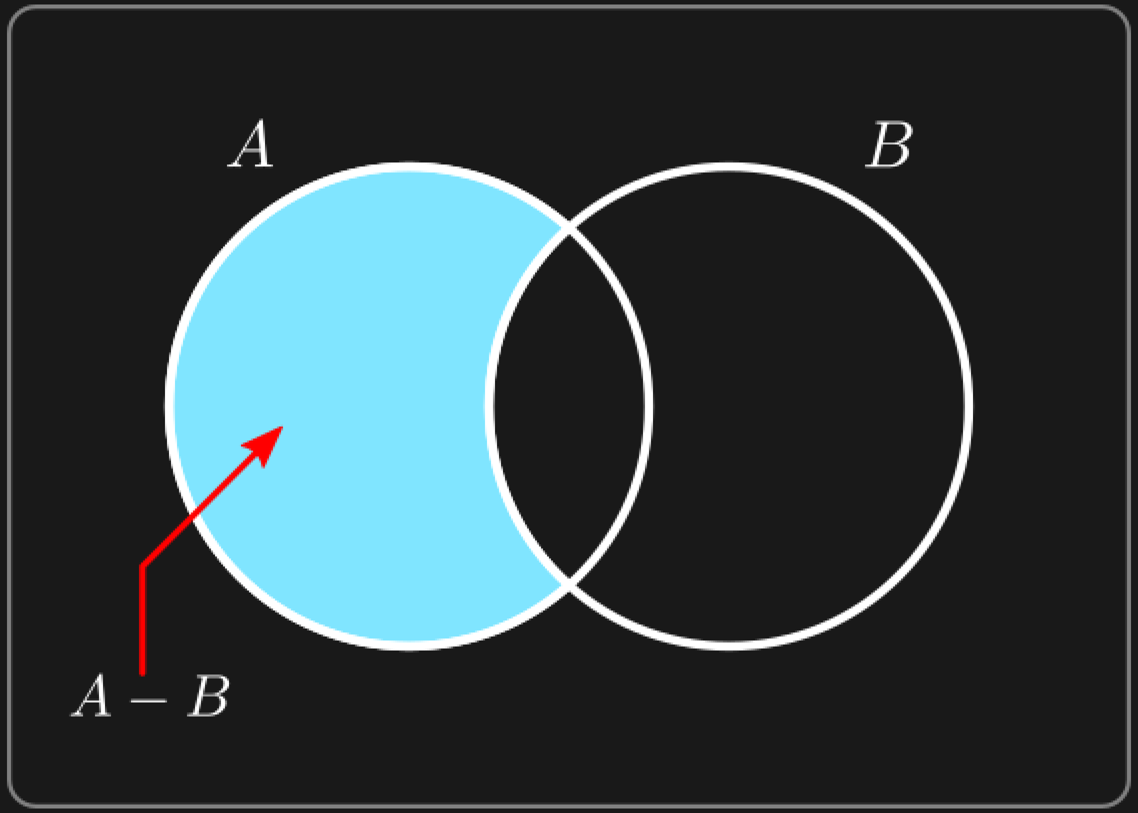


ANOTHER OPERATION

DEFINITION:

Let A and B be sets. Then the **difference**, $A - B$ is

$$A - B = \{x \in A \ : \ x \notin B\}$$



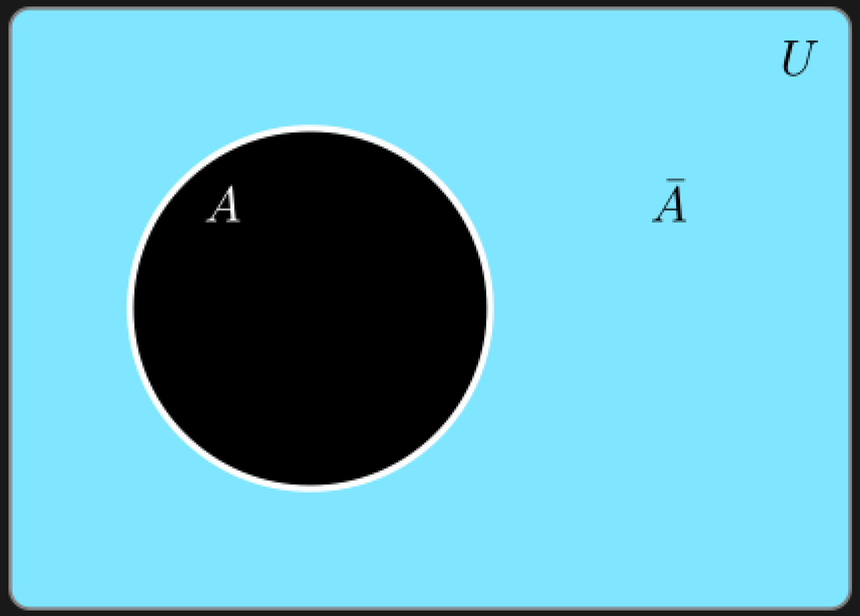
UNIVERSALS AND COMPLEMENTS

DEFINITION:

Given a universal set U and $A \subset U$, the complement of A is

$$\bar{A} = \{x \in U \ : \ x \notin A\} \qquad \text{or equivalently} \qquad x \in \bar{A} \iff x \notin A$$

The universal set is the set from which we draw elements in the current *context*



NOTES AND EXAMPLES

- Compl-e-ment vs Compl-i-ment
- $A - B$ also written $A \setminus B$ and is called the **relative complement of** B in A
- We have $A - B = A \cap \bar{B}$
- Let $U = \mathbb{N}$, $A = \{1, 2, 3, 4\}$, $B = \{p \ : \ p \text{ is prime}\}$ and $C = \{4, 5, 6, 7\}$

$$A - C = \{1, 2, 3\} \qquad A - B = \{1, 4\} \qquad \bar{A} = \{n \in \mathbb{N} \ : \ n \geq 5\}$$

ORDERED PAIRS

Sets don't care about order, but many applications need *pairs* of objects.

DEFINITION:

An **ordered pair** of elements is an *ordered* list of two elements.

The ordered pair of two elements a, b is written (a, b) and satisfies

- $(a, b) = (c, d)$ only when $(a = c)$ and $(b = d)$, and
- $(a, b) \neq (b, a)$ unless $(a = b)$.

Warning

- use correct notation: $\{1, 3\}$ is a set, $(1, 3)$ is an ordered pair
- we sometimes use $(1, 3) = \{x \in \mathbb{R} \ : \ 1 < x < 3\}$ — give your **reader** context

CARTESIAN PRODUCT

DEFINITION: CARTESIAN PRODUCT.

The Cartesian product of sets A, B is

$$A \times B = \{(a, b) \ : \ a \in A, b \in B\}$$

Note for $A, B \neq \varnothing$, $A \times B \neq B \times A$ unless $A = B$.

Let $A = \{a, b, c\}$ and $B = \{1, 2\}$ then

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$$