# Worksheet for Week 5

**Definition:** Let $L \in \mathbb{R}$. We say that a sequence $(x_n)$ converges to $L$, denoted by $x_n \to L$ iff,

$$\forall \epsilon > 0, \exists N \in \mathbb{N}, \text{ s.t. } \forall n \geq N, \ |x_n - L| \leq \epsilon.$$

1. Show that $(x_n) = \left(\frac{n}{n^2+1}\right)$ converges to 0.

   Hint: For $n \in \mathbb{N}$, we have $\frac{n}{n^2+1} < \frac{1}{n}$.

2. Show that $(x_n) = \left(\frac{n}{\sqrt{n^2+1}}\right)$ doesn't converge to 0.

   Hint: What does it mean for a sequence to NOT converge to a number?

3. Let $(x_n), (b_n)$ be sequences. Prove that if $0 < x_n < b_n \ \forall n \in \mathbb{N}$ and $b_n \to 0$, then $x_n \to 0$.

   Hint: If $0 < x_n < b_n$, the $|x_n| < |b_n|$. Thus, if we can make $|b_n| < \epsilon$, then we will have $|x_n| < \epsilon$.

**Definition:** Let $A$ be a set and $L \in \mathbb{R}$. We say that a function $f : A \to \mathbb{R}$ has the limit $L$ as $x$ goes to $a$, denoted by $\lim_{x \to a} f(x) = L$, if

$$\forall \epsilon > 0, \exists \delta > 0, \text{ s.t. } (0 < |x - a| < \delta) \implies (|f(x) - L| < \epsilon).$$

4. Show that $\lim_{x \to 1}(5x + 3) = 8$.

   Hint: Be careful that $\delta$ may depend on $\epsilon$.

5. Prove that $\lim_{x \to 2} \left(\frac{1}{x}\right) = \frac{1}{2}$.

   Hint: You may need to have more than one condition on $\delta$.

6. Let $f, g$ be functions and $L_1, L_2 \in \mathbb{R}$. Show that if $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$, then $\lim_{x \to a}(f+g)(x) = L_1 + L_2$.

   Hint: Triangle inequality and a good scratchwork will be crucial.

Before next week, watch videos 18 and 19 in `https://personal.math.ubc.ca/~PLP/auxiliary.html`.