Worksheet for Week 8

1. Let A, B and C be sets. Then prove or disprove: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
2. Let A, B and C be sets. Then prove or disprove: $A - (B - C) = (A - B) - C$.

   Hint: Venn diagrams will be useful here.
3. If A and B are sets, then prove that $(A \subseteq B) \implies (\mathcal{P}(A) \subseteq \mathcal{P}(B))$.

   Hint: Remember that subset relation is a conditional statement.
4. (Old final question) Let A, B, C be sets. Prove that $A \times C \subseteq B \times C$ if and only if $A \subseteq B$ or $C = \emptyset$.

   Hint: For one of the directions, recall $(P \implies (Q \vee R)) \equiv (P \implies ((\sim Q) \implies R))$.
5. If A and B are sets, then prove that $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

Before the following examples, watch videos 26 and 27 in `https://personal.math.ubc.ca/~PLP/auxiliary.html`.

6. Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation R that expresses "$\nmid$" (does not divide) on A as a set of ordered pairs.

   Note: We need to make sure that we are not leaving any one of the ordered pairs out. If we do, it is not the same relation anymore.
7. (Old final) Determine which of the following relations, $\mathbf{R}$, are reflexive, symmetric and transitive on the given set A. (We call a relation that satisfies all 3 properties, an equivalence relation.) Prove your answers.

   (a) $\mathbf{R} = \{(x, y) \in \mathbb{R} \times \mathbb{R} \colon y = x^2\}$ on $A = \mathbb{R}$.

   (b) $\mathbf{R} = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1)\}$ on $A = \{1, 2, 3\}$.

   (c) $\mathbf{R} = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \colon 3 \text{ divides } a - b\}$ on $A = \mathbb{Z}$.
8. Define a relation on $\mathbb{Z}$ as aRb if $3 \mid (2a - 5b)$. Is R reflexive, symmetric, transitive? Justify your answer.
9. Let $\mathcal{R}$ and $\mathcal{R}'$ be two relations on the same set A. Prove or disprove the following.

   (a) If $\mathcal{R}$ and $\mathcal{R}'$ are transitive, then the relation $\widehat{\mathcal{R}}$ defined as $\widehat{\mathcal{R}} = \mathcal{R} \cup \mathcal{R}'$ is transitive.

   (b) If $\mathcal{R}$ and $\mathcal{R}'$ are transitive, then the relation $\widehat{\mathcal{R}}$ defined as $\widehat{\mathcal{R}} = \mathcal{R} \cap \mathcal{R}'$ is transitive.

Before the following week, watch video 28 in `https://personal.math.ubc.ca/~PLP/auxiliary.html`.