

MATH 100 Section 108 – 2019W

In-Class Problem Sheet 2

A. Alperen Bulut

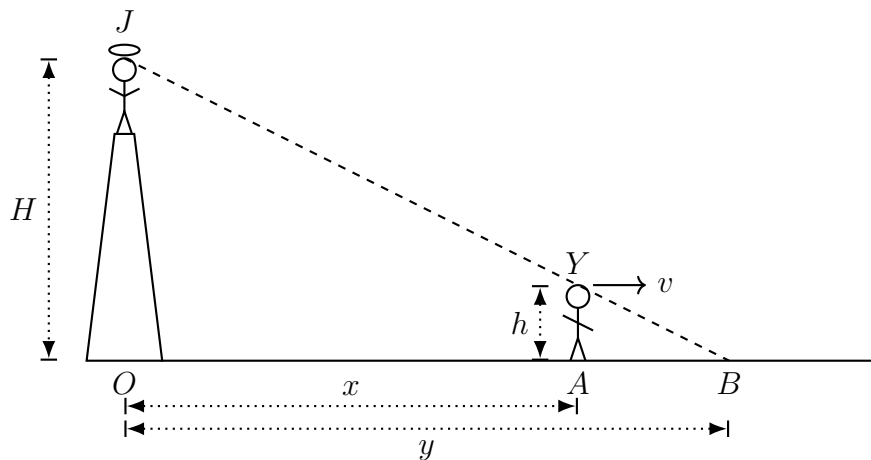
October 28, 2019

1 Related Rates Continued

Problem 1.1 (Jesus illuminating Yeezus, who is illuminating us). Suppose Jesus is on top of a hill so that He is 60 ft above from the ground. Kanye West, who is 6 ft, is on the ground, being illuminated with His divine knowledge. The horizontal distance between Jesus and Yeezus is 50 ft, and Yeezus is walking away from Jesus with a speed of 5 ft/s so that he can spread the Gospel.

We, the mortals on the ground, can gain this wisdom only after it reaches Ye's mind and gets converted into a gospel record such as *Jesus is King*. Assume that the divine knowledge behaves just like light (of course). Then how fast is the divine knowledge (or, the shadow of Ye's head) moving on the ground?

Hint: Consider the following diagram.



Answer: 50/9 ft/s.

2 Linear Approximation $[f(x) \approx L(x) = f(a) + f'(a)(x - a)]$

Problem 2.1 (A Real Story). When I, AAB, was a sophomore in undergrad, yours truly was taking a course on materials science and engineering. Being the complete idiot I am when it comes to the “real world,” I of course forgot to bring my calculator to the midterm, and realized this only thirty seconds before the exam started.

One of the multiple-choice problems was about finding the ratio of electrons that manage to penetrate a slab of material, and I found the answer as

$$(0.998)^{10}.$$

However, all the choices were in decimals with two significant figures.

Realizing the deep mess I’m in and having no way out, I found the situation extremely funny. I decided to improvise, because I had nothing to lose but my pride. I knew Calculus, and I realized I could approximate functions. I thought linear approximation was the best I could do at that moment.

What did I do, and what was the correct answer I found?

Answer: 0.98. Note that the correct value of $(0.998)^{10}$ is about 0.98017904335.

Problem 2.2. Approximate $\sqrt{8.9}$ using a linear approximation.

Answer: $3 - 1/60 = 2.98\bar{3}$. Note that the correct value of $\sqrt{8.9}$ is about 2.98328677804.

Problem 2.3. Approximate $\sin(3)$ using a linear approximation. It’s OK to have π in your answer.

Answer: $\pi - 3$, which is about 0.14159265359. Note that the correct value of $\sin(3)$ is about 0.14112000806.

Problem 2.4. Approximate $e^{1/10}$ using a linear approximation.

Answer: 1.1. Note that the correct value of $e^{1/10}$ is about 1.10517091808.

3 Quadratic Approximation

$$\left[f(x) \approx P(x) = f(a) + f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2 \right]$$

Problem 3.1. Approximate $\log(1.1)$ using a quadratic approximation.

Answer: 0.095. Note that the correct value of $\log(1.1)$ is about 0.0953101798.

Problem 3.2. Approximate $\sqrt[3]{28}$ using a quadratic approximation.

Answer: $3 + \frac{1}{3^3} - \frac{1}{3^7}$.

Problem 3.3. Determine what $f(x)$ and a should be used so that you can approximate the following using a quadratic approximation.

(a) $\log(0.9)$

(b) $e^{-1/30}$

(c) $\sqrt[5]{30}$

(d) $(2.01)^6$

Answer: Defeats the purpose.