Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. How many five-digit positive integers have two 2's and three 3's in their decimal representation?

1. _____

2. There are four different positive integers a, b, c, and d such that

$$a^3 + b^3 = c^3 + d^3 = 1729.$$

What is the value of $a + b + c + d$?

2. _____

3. What is the largest number of $20 \times 20 \times 10$ wooden boxes that can be put in a $50 \times 50 \times 40$ box? All measurements are in centimetres. Boxes cannot be cut, or changed in shape or size.

3. _____ boxes

4. Some positive integers can be expressed as the sum of two perfect squares. For example, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$, $4 = 0^2 + 2^2$, $5 = 1^2 + 2^2$. But, for example, 3, 6, and 7 cannot be expressed as the sum of two perfect squares. How many of the integers from 1 to 25 *cannot* be expressed as the sum of two perfect squares?

4. _____

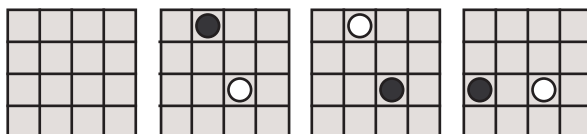5. In the 3×3 grid below, every point is at a unit distance from its nearest horizontal or vertical neighbours. Call a line segment *good* if the endpoints of the line segment are grid points, and the line segment contains no grid point other than its endpoints. (Two good line segments have been drawn in the diagram.) How many different good line segments are there?

5. _____ segments
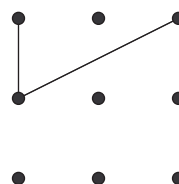
6. The picture below (on the left) shows a 4×4 "checker board." How many ways are there to place a white piece and a black piece on two different squares of this board, so that the two pieces are *not* on squares that are next to each other. (Two squares are considered next to each other if they have a side or a vertex in common.) The three pictures below (on the right) show three of the allowed ways to place the white piece and the black piece.

6. _____ ways

7. Let N be a positive integer. Define $f(N)$ to be the total number of occurrences of primes in the prime factorization of N. For example, $f(60) = 4$, since 60 is the product of the primes 2, 2, 3, and 5. How many positive integers $N \leq 125$ are there such that $f(N) = 4$?

7. _____

8. In the 4×4 grid below, every point is at a unit distance from its nearest horizontal or vertical neighbours. How many ways are there to choose a collection of 4 points of the grid so that these 4 points form the corners of a square?

8. _____ ways

9. Imagine listing, in increasing order, the positive integers which are neither perfect squares nor perfect cubes, nor perfect fourth powers, nor perfect fifth powers, and so on. The first few numbers in the list are 2, 3, 5, 6, 7, and 10. What is the 250-th number in the list?

9. _____

10. The figure below is a quarter-circle with center O and radius 4. The point M is on the curved part of the quarter-circle, and the length of the arc PM is equal to the length of the arc MQ. The point N bisects the line segment OP. What is the area of the shaded region? Express your answer in terms of π.

10. _____ units^2