

REGIONAL 2010 FACE-OFF
FIRST 40 QUESTIONS AND ANSWERS

1. What is 150% of 2010? 1. 3015

2. What is the value of $\frac{4^4}{8^2}$? 2. 4

3. What is the largest prime that divides $5! + 1$? 3. 11

4. What is the smallest positive integer which is simultaneously a multiple of 20, 25, and 30? 4. 300

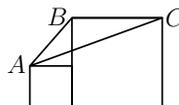
5. If $6x + 10 = 101$, what is the value of $12x + 10$? 5. 192

6. What is the value of $\frac{3^5 - 3^3}{2^3}$? 6. 27

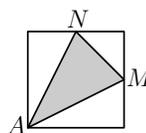
7. An accurate 12-hour clock shows that it is exactly 12:00. What time shows on the clock 600 seconds later? Express your answer in the usual hours:minutes format. 7. 12:10

8. Evaluate 15% of the reciprocal of 0.005. 8. 30

9. In the picture below, the smaller square has side 7 and the larger square has side 15. What is the area of $\triangle ABC$? 9. 60

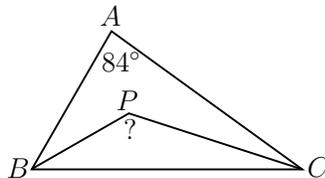


10. How many integers between 1 and 100 are divisible by 6? 10. 16
11. How many ordered pairs (a, b) are there such that a and b are integers (not necessarily positive) and $|a| + |b| \leq 1$? 11. 5
12. Alicia sold her condo for 20% more than she paid for it, and made a gross profit of \$90,000. For how many dollars did she sell her condo? 12. 540,000 (dollars)
13. How many four-letter “words” are there which use only the letters A and/or B, and in which there are no consecutive occurrences of B? (For example, AAAA and ABAB qualify, but ABBA does not.) 13. 8
14. There is an integer N such that $N^3 = 6859$. What is the value of N ? 14. 19
15. The points M and N are the midpoints of two adjacent sides of a 1×1 square. What is the area of the shaded triangle? Give the answer as a common fraction. 15. $\frac{3}{8}$



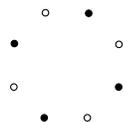
16. Evaluate $(6 \times 12 \times 24)^{1/3}$. 16. 12
17. Express $\left(1 - \frac{1}{3}\right) + \left(\frac{1}{9} - \frac{1}{27}\right)$ as a common fraction. 17. $\frac{20}{27}$

18. On Monday, 8 of the 10 students in the math class took a test. Their mean score was 60. On Tuesday, the remaining 2 students took the test. Their mean score was 95 (they had seen the questions). What was the mean class score on the test? 18. 67
19. Three standard dice are tossed. What is the probability that the sum is less than 3? 19. 0
20. A lidless $3 \times 3 \times 3$ box is completely filled with $1 \times 1 \times 1$ cubes. How many of the $1 \times 1 \times 1$ cubes touch a side or the bottom of the box? 20. 25
21. What is the measure, in degrees, of the acute angle between the hour hand and the minute hand of a clock at 3:30? 21. 75 (degrees)
22. The sum of 5 consecutive odd numbers is 55. What is the largest of the 5 numbers? 22. 15
23. The surface area of a cube is $\frac{3}{2}$ square metres. What is the volume of the cube, in cubic metres? Express the answer as a common fraction. 23. $\frac{1}{8}$ (square metres)
24. What is the value of $\frac{(4!)(6!)}{5!}$? 24. 144
25. In $\triangle ABC$, the angle at A is 84° . The bisectors of the angles at B and C meet at P . How many degrees are in the measure of $\angle BPC$? 25. 132 (degrees)



26. A list of 10 numbers has a mean of 50. One number is added to the list, and the mean of the new list of 11 numbers is 49. What number was added to the list? 26. 39

27. The 8 points below represent the vertices of a regular octagon. These vertices are alternately painted red and blue. How many lines are there that contain a red point and a blue point? 27. 16



28. Simplify $\frac{2^{-3} + 3^{-3}}{6^{-3}}$. 28. 35

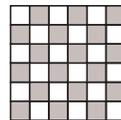
29. Suppose that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$. If $y = 10$ and $z = 8$, what is the value of x ? 29. 40

30. What is the value of $2010^2 - 2009^2$? 30. 4019

31. Suppose that a and b are integers and $2^a - 2^b = 16$. What is the value of $a + b$? 31. 9

32. A line passes through the points $(-1, 2)$, $(2, -1)$, and $(x, -5)$. What is the value of x ? 32. 6

- 33.** A penny is placed on one of the squares of the 6×6 “chessboard” below, and then a dime is placed on a randomly chosen square different from the square occupied by the penny. What is the probability that the dime is neither in the same row nor in the same column as the penny? Express your answer as a common fraction.



33. $\frac{5}{7}$

- 34.** Simplify:

$$(2^{10} + 2^{10} + 2^{10} + 2^{10})^{1/3}$$

34. 16

- 35.** How many positive integers between 1 and 30 (inclusive) are divisible by 3 or by 5 or by both?

35. 14

- 36.** Alfie wrote four tests, each out of 100. His average over the four tests is 76. His average on the last two tests is 10 more than his average on the first two tests. What is his average on the first two tests?

36. 71

- 37.** Express $\frac{\sqrt{8} - \sqrt{2}}{\sqrt{8} + \sqrt{2}}$ as a common fraction.

37. $\frac{1}{3}$

- 38.** Alicia has 5 dollars less than Beti, and Cecille has as much money as Alicia and Beti have between them. Altogether, the three people have a total of 270 dollars. How many dollars does Alicia have?

38. 65 (dollars)

- 39.** Evaluate $(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!)$.

39. 119

- 40.** Suppose that $f(n+2) = f(n) + f(n+1)$ for every positive integer n . Given that $f(1) = 1$ and $f(3) = 48$, what is the value of $f(2)$?

40. 47