

REGIONAL 2011 FACE-OFF
QUESTIONS AND ANSWERS

1. How many integers between 1 and 100 are divisible by 7?

1. Answer: 14 (integers)

2. Express $\frac{\frac{1}{3} - \frac{1}{12}}{\frac{1}{3} + \frac{1}{12}}$ as a common fraction.

2. Answer: $\frac{3}{5}$

3. It takes 450 seconds for 4 teenagers to eat 3 pizzas. If all teenagers always eat at the same constant rate, how many seconds does it take 3 teenagers to eat 4 pizzas?

3. Answer: 800 (seconds)

4. What is the smallest prime number which is larger than 139?

4. Answer: 149

5. If $3x + 10 = 49$, what is the value of $6x + 10$?

5. Answer: 88

6. In a bag of jelly beans, $\frac{1}{5}$ are white, $\frac{1}{5}$ are yellow, $\frac{1}{4}$ are green, $\frac{1}{4}$ are red, and the remaining 12 are black. How many red jelly beans are in the bag?

6. Answer: 30 (red ones)

7. The temperature F in degrees Fahrenheit is related to the temperature C in degrees Celsius by the equation $F = \frac{9}{5}C + 32$. If the temperature of an oven in degrees Fahrenheit is 320, what is the temperature of the oven in degrees Celsius?

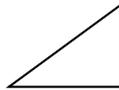
7. Answer: 160 (degrees Celsius)

8. How many ordered pairs (a, b) of positive integers are there such that $a + b \leq 5$?

8. **Answer:** 10 (ordered pairs)

9. The measures of the acute angles of a right triangle are in the ratio 2:3. What is the degree measure of the smallest angle of the triangle?

9. **Answer:** 36 (degrees)



10. The first few terms of a sequence are

1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1,

and the obvious pattern continues forever. What is the product of the 66-th term and the 67-th term of the sequence?

10. **Answer:** 12

11. Two fair dice are tossed. What is the probability that the sum is 6? Express the answer as a common fraction.

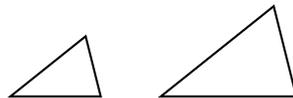
11. **Answer:** $\frac{5}{36}$

12. The number $1\frac{6}{7}$ is how many percent of $2\frac{6}{7}$?

12. **Answer:** 65 (percent)

13. Two similar triangles have area 4 square units and 9 square units respectively. If the perimeter of the smaller triangle is 10 units, how many units are in the perimeter of the larger triangle?

13. **Answer:** 15 (units)



14. On a planet far far away, a year is 500 days and a week is 7 days, with the usual English names. New Year's day this year was on a Sunday. On what day of the week is New Year's day next year?

14. Answer: Wednesday

15. An answering machine can hold up to 10 minutes of messages. Any message uses up at least 12 seconds. What is the largest number of messages that the machine can hold?

15. Answer: 50 (messages)

16. When the bowl of mixed nuts was set out, it was (by weight) 45% peanuts, 25% almonds, 20% cashews, and 10% hazelnuts. Alphonse picked out all the almonds and ate them. What percent (by weight) of the nuts in the bowl are now peanuts?

16. Answer: 60 (percent)

17. In a school election, every one of the 2011 students voted for exactly one of A or B. If A got 1100 votes, how many more votes than B did A get?

17. Answer: 189 (more votes)

18. Let $H(x, y) = \frac{2xy}{x + y}$. What is the value of $H(7, 42)$?

18. Answer: 12

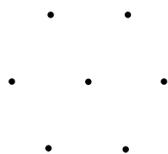
19. American Thanksgiving is on the fourth Thursday in November. What day of the month is the last possible day for American Thanksgiving? Your answer should be a number between 1 and 30.

19. Answer: 28

20. Two-thirds of a group of students are dark-haired, one-quarter are blonde, and the remaining 30 students are red-haired. How many of the students in the group are *not* red-haired?

20. Answer: 330 (students)

21. The figure below consists of 7 points, namely the 6 vertices of a regular hexagon and the centre of that hexagon. How many triangles have one vertex at the centre of the hexagon, and the other two vertices at vertices of the hexagon?



21. **Answer:** 12 (triangles)

22. What is the largest prime that divides 9999?

22. **Answer:** 101

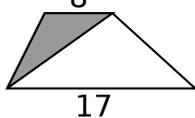
23. A pack of 6 cans of cola costs \$4.50, plus 12% tax, plus a 5 cent deposit per can. How much does one have to pay for a pack of 6 cans? Give the answer in dollars, to the nearest cent.

23. **Answer:** 5.34 (dollars)

24. How many whole numbers are there between $\sqrt{88}$ and $\sqrt{888}$?

24. **Answer:** 20 (whole numbers)

25. What common fraction of the area of the trapezoid is shaded?



25. **Answer:** $\frac{8}{25}$

26. X has 60% as much money as Y. Between them they have \$1000. How many dollars does X have?

26. **Answer:** 375 (dollars)

27. Given that $(x - y)^2 = 39$ and $xy = 40$, what is the value of $(x + y)^2$?

27. **Answer:** 199

28. What is the smallest positive integer which is divisible by 12 and whose decimal representation contains no digits other than 0's and 1's?

28. Answer: 11100

29. Given that $\sqrt{1 + \sqrt{4 + x}} = 3$, what is the value of x ?

29. Answer: 60

30. What is the area, in square cm, of the isosceles triangle that has sides 2, 4, and 4 cm? Express the answer in simplest radical form.



30. Answer: $\sqrt{15}$ (square cm)

31. The lengths of the three sides of a triangle are in the ratio 4:5:6. The triangle has perimeter 180 cm. How many cm are in the length of the longest side of the triangle?

31. Answer: 72 (cm)

32. Four fair coins are tossed. What is the probability of getting at least one head and at least one tail? Express the answer as a common fraction.

32. Answer: $\frac{7}{8}$

33. What is the smallest positive integer that cannot be expressed as the difference between two prime numbers?

33. Answer: 7

34. The top 100 rock and roll songs of all time are played one after the other. Each song lasts 3 minutes, and there is a 1 minute commercial break between songs. From the start of the first song to the end of the last song takes 6 hours plus how many minutes?

34. Answer: 39 (minutes)

35. A basketball team has 7 wins and 13 losses. To reach the playoffs it must win at least 50% of the regular-season games that it plays. What is the largest number of the remaining 35 regular season games that the team can lose and still reach the playoffs?

35. Answer: 14 (games)

36. If $4^{(4^4)} = 2^{(2^y)}$, what is the value of y ?

36. Answer: 9

37. The first term of an arithmetic sequence is 1, and the sum of the first 4 terms of the sequence is 100. What is the sum of the first 5 terms of the sequence?

37. Answer: 165

38. What is the smallest whole number N such that $75N$ is a perfect cube?

38. Answer: 45

39. How many positive integers between 1 and 60 (inclusive) are divisible by 3 or by 5 or by both?

39. Answer: 28 (integers)

40. What is the largest perfect square which is less than 3^8 ?

40. Answer: 6400

41. What is the smallest perfect square which is in the arithmetic sequence 1, 6, 11, and also in the arithmetic sequence 7, 13, 19, ?

41. Answer: 121

42. How many integer values of n are there such that $|(6n-77)(n+30)|$ is a prime?

42. Answer: 3 (integer values)

43. How many 3-digit positive integers are there all of whose digits are odd?

43. Answer: 125

44. Alphonse took the same algebra test a total of 3 times. Each time he retook the test, the number of questions he answered correctly increased by 50%. If on the last test he got 36 of the 70 questions right, how many questions did he get right the first time he took the test?

44. Answer: 16 (questions)

45. What is the value of $\frac{\sqrt{b^2 - 4ac} - b}{2a}$ when $a = 1$, $b = 3$, and $c = -4$?

45. Answer: 1