Provincial 2012 Face-off
Questions and Answers

1. Eighty-eight is 16% of what number?
1. Answer: 550

2. If $x^2 = \dfrac{9}{256}$, and x is positive, what is the value of x? Express the answer as a common fraction.
2. Answer: $\dfrac{3}{16}$

3. How many *real numbers* (they are not necessarily integers) x are there such that $x > 1$ and $\dfrac{8}{x}$ is an integer?
3. Answer: 7

4. What is the smallest positive integer n such that $8^n > 2^{22}$?
4. Answer: 8

5. What is the value of

$$1 + 2 + 3 + \cdots + 13 + 14 + 15 + 14 + 13 + \cdots + 3 + 2 + 1?$$

5. Answer: 225

6. What is the value of $\dfrac{\frac{1}{2} - \frac{1}{5}}{\frac{1}{10}}$?
6. Answer: 3

7. How many perfect squares are there between 11 and 111?
7. Answer: 7

8. A very long test has 99 questions, numbered 1 to 99. The test is 9 pages long, and each page has the same number of questions. What is the number of the fifth question on the fifth page?
8. Answer: 49

9. Of the 20 marbles in a bag, 10 are red and 10 are green. Two marbles are removed from the bag. What is the probability that these two marbles are of different colours? Express the answer as a common fraction.
9. Answer: $\dfrac{10}{19}$

10. The average class size in the year 2000 was 27. Now the average class size is 31. By how many percent has the average class size increased from the year 2000 to now? Round the answer to the nearest integer. Thus an answer like 17 is of the right shape.
10. Answer: 15 (percent)

11. Square $\mathcal{A}$ has area 4 and square $\mathcal{B}$ has area 9. How many percent is the *side* of square $\mathcal{B}$ greater than the side of square $\mathcal{A}$?
11. Answer: 50

12. A fair coin is tossed 4 times in a row. What is the probability of getting a total of 3 heads (in any order) and 1 tail? Express the answer as a common fraction.
12. Answer: $\dfrac{1}{4}$

13. How many positive integers n are there such that $\dfrac{1024}{n}$ is a perfect square?
13. Answer: 6

14. If $9^2 \times 27^3 = 3^n$, what is the value of n?
14. Answer: 13

15. The operation $\circledast$ is defined by the rule

$$x \circledast y = x^2 - 2xy + y^2.$$

What is the value of $6 \circledast (-4)$?
15. Answer: 100

16. The three cans of cola that Alphonse drinks every day together supply 20% of Alphonse's daily caloric requirements, which are 2100 calories. How many calories are in 1 can of cola?
16. Answer: 140

17. If $x^2 = 49$, what is the sum of the two possible values of $(x+1)^2$?
17. Answer: 100

18. Alicia and Beti drive separately from A to B. The distance from A to B is 200 km. They start at the same time. Alicia averages 100 km/hour, and Beti averages 75 km/hour. How many minutes after Alicia arrives at B does Beti arrive at B?
18. Answer: 40(minutes)

19. How many 3-digit positive integers are there all of whose digits are all distinct and odd? Note that 795 is such a number, but 757 is not.
19. Answer: 60

20. How many integers from 1 to 1000 are perfect squares and have the sum of their decimal digits equal to 9?
20. Answer: 8

21. Let $N = (201)^2$. What is the sum of all the decimal digits of N?
21. Answer: 9

22. How many integers between 1 and 1000 are divisible by both 6 and 16?
22. Answer: 20

23. Simplify $\dfrac{2^{-3} + 3^{-3}}{6^{-3}}$.
23. Answer: 35

24. Suppose that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$. If $y = 16$ and $z = 20$, what is the value of x?
24. Answer: -80