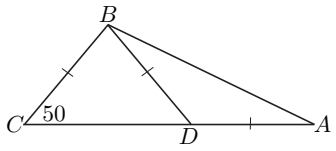
Regional 2007 Face-off Questions

1. Alfie walked 0.6 km in 12 minutes. At this rate, how many metres can Alfie walk in 30 minutes? 1.

2. Express $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$ as a common fraction. 2.

3. Two standard dice are tossed. What is the probability that the product of the two numbers obtained is 5? Express the answer as a common fraction. 3.

4. In the diagram below, $BC = BD = DA$ and $\angle BCA = 50°$. What is the degree measure of $\angle ABC$? 4.

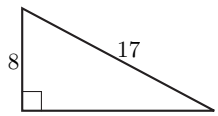


5. How many positive integers less than 2007 are divisible by both 30 and 100? 5.

6. The sum of three different positive integers is equal to 10. What is the largest possible value of the sum of their squares? 6.

7. You are told that 1 is halfway between x and 17. What is the value of x? 7.

8. The triangle below is right-angled. The hypotenuse has length 17 cm and one leg has length 8 cm. What is the number of cm^2 in the area of the triangle? 8.



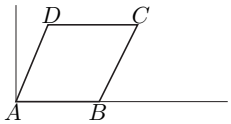
9. Let x be the 2007-th term of the geometric sequence 3, 6, 12, ..., and let y be the 2004-th term of the same sequence. What is the value of x/y? 9.

10. Fifty percent of 50% of a certain number is 50. What is the number? 10.

11. Suppose that N and $N + 17$ are both perfect squares. What is the value of N? 11.

12. Two prime numbers p and q add up to 70. What is the smallest possible positive value of $p - q$? 12.

13. The figure $ABCD$ below is a rhombus, with the vertices A, B, C, and D going counterclockwise. Vertex A has coordinates $(0, 0)$, vertex B is on the positive x-axis, and vertex D has coordinates $(5, 12)$. What is the area of the rhombus? 13.



14. A circle has radius $\frac{2007}{\pi}$. What is the circumference of the circle? 14.

15. Suppose that $\frac{x}{a} = \frac{a}{5}$ and $a = 20$. What is the value of x? 15.

16. A basketball team won 20% more games than it lost. The team played 88 games. How many games did it win? (Ties are impossible in basketball.) 16.

17. What is the smallest positive integer n such that n is a multiple of 5, $n + 1$ is a multiple of 3, and $n + 2$ is a multiple of 2? 17.

18. The product of two consecutive odd integers is 255. What is the sum of the two odd integers? 18.

19. What is the sum of all the positive factors of 62? 19.

20. One foot is equal to 12 inches. Alan is jogging at 600 feet per minute. What is his speed in inches per second? 20.

21. The sum of two numbers is 100 and one of the numbers is 17. What is the positive difference between the two numbers? 21.

22. One corner of a square is at $(1, 2)$. The diagonally opposite corner is at $(7, 8)$. How many square units are in the area of the square ? 22.

23. Express $\dfrac{10! - 9! - 8!}{10! + 9! + 8!}$ as a common fraction. 23.

24. What is the value of 2007^2 ? 24.

25. If $3x - 2 = \dfrac{1}{7}$, what is the value of x ? 25.

26. An equilateral triangle has side 4. What integer is closest to the area of the triangle ? 26.

27. Simplify: $\sqrt{\dfrac{1}{16} + \dfrac{3}{50}}$. 27.

28. Suppose that x and y are positive integers and $x^2 - y^2 = 64$. What is the smallest possible value of $x + y$? 28.

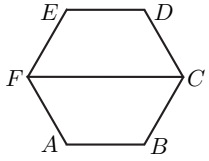
29. The surface area of a cube is 216 cm^2. What is the number of cm^3 in the volume of the cube? 29.

30. What is the value of $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + 5^3}$? 30.

31. What is the remainder when 2^{2007} is divided by 9 ? 31.

32. Alphonse's hourly wage is 15% less than Beth's. Gamal's hourly wage is 2% greater than Beth's. How many percent is Gamal's wage greater than Alphonse's ? 32.

33. The perimeter of the regular hexagon $ABCDEF$ is 192 cm. How many cm are in the perimeter of the trapezoid $ABCF$? 33.



34. What is the remainder when 1111 is divided by 7 ? 34.

35. What is the value of 35.

$$1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \cdots + 70 + 71 - 72 + 73 + 74 - 75\,?$$

36. The sum of the squares of two positive integers is equal to 73. What is the sum of the two integers ? 36.

37. The opera consists of 2 acts of equal length with an intermission in between. The length of the opera (including the 30 minute intermission) is exactly 3 hours. What is the ratio of the length of the intermission to the length of the first act? Express as a common fraction. 37.

38. What is the smallest positive integer n such that $5^n > 4^{n+1}$? 38.

39. What is the value of $(\sqrt{4} + \sqrt{2})^2 + (\sqrt{4} - \sqrt{2})^2$? 39.

40. Solve for x: $(4^5)^6 = (2^x)^3$. 40.