List 3 of Questions from 2006 Provincial Competition

From Bull's Eye Stage

1. Assume that the Earth is a sphere of radius 6400 km, and that Mont Blanc, the highest mountain in the Alps, is 4.8 km high. If we make a scale model of the Earth of radius 0.20 metres, how many millimetres high should Mont Blanc be on the model? Give the answer as a decimal, correct to 2 decimal places.

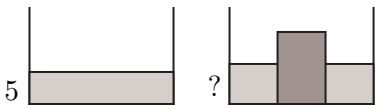
   1. _____ mm

2. The first term of an arithmetic sequence is 1 and the last term is 4. The sum of all the terms is 30. What is the second term? Express your answer as a common fraction.

   2. _____

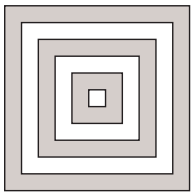
3. A tall cylindrical cooking pot has a 12 cm inner base radius, and has some water in it; the depth of the water is 5 cm. A tall heavy closed cylindrical can is placed in the pot, with one of the flat sides down. The base radius of the can is 4 cm. How many cm deep is the water in the pot now? Give the answer correct to 3 decimal places.

   3. _____ cm



4. What is the shaded area in the diagram below? The six squares have sides of length 1, 3, 5, 7, 9, and 11 units.

   4. _____ units^2



From Blitz Stage

1. Alfie spends one-third of his allowance on books and two-thirds on (healthy) snacks. Suppose the price of books goes down by 8% and the price of snacks goes up by 10%. What percent increase in allowance should Alfie get so that he can keep on buying as many books and as many snacks as before prices changed?

   1. _____ %

2. In the country of Decima, instead of dividing the usual clock into 12 hours, they divide it into 10 equal parts. What time does an ordinary Canadian clock show when a Decima clock shows 8.00? Give your answer in the usual hours : minutes format.

   2. _____

3. Together, A and B own 64 DVDs (that is, the number of DVDs owned by A plus the number of DVDs owned by B is 64). Together, B and C own 81 DVDs. And together, C and D own 100 DVDs. How many DVDs do A and D own together?

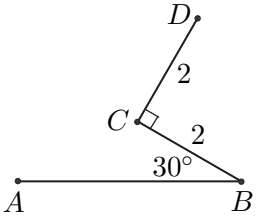
   3. _____ DVDs

4. Using the letters a, b, c, d, and e, we can form 625 four-letter "words." Suppose we list these words in alphabetical order. The first six words are aaaa, aaab, aaac, aaad, aaae, and aaba. What is the 235-th word in the list? 4. _____

5. What is the smallest positive integer n such that $10n + 1$ is a power of 7? 5. _____

6. The integers from 1 to 24 are written on index cards, one number to each card. Alicia picks a card at random. Let x be the probability that the number on her card is divisible both by 2 and by 3, and let y be the probability that the number is divisible by 2 or by 3 (or both). What is $\frac{x}{y}$? Express your answer as a common fraction. 6. _____

7. In the figure below, $\angle ABC$ has measure 30°, $\angle BCD$ is a right angle, and $BC = CD = 2$. What is the (perpendicular) distance from D to the line AB? Give the answer in simplest radical form. 7. _____ units



8. A bowl contains 600 slips of paper. Each slip has one of the six letters A, B, C, D, E, or Q written on it, and there are 100 of each kind. How many slips must you grab in order to be sure that among the slips you grab there is at least 1 A, or at least 2 B's, or at least 3 C's, or at least 4 D's, or at least 5 E's? 8. _____ slips

9. Xavier has 5 friends, A, B, C, D, and E. He wants to have dinner with 1 or more of these. Unfortunately, A and B dislike each other and cannot both be invited to the same dinner. In how many ways can Xavier select the people he will have dinner with? 9. _____ ways

From Co-op Stage

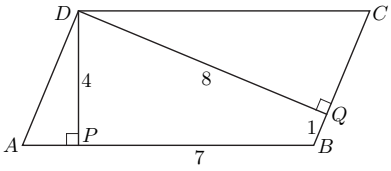
1. The integers from 1 to 5 are written on index cards, one number to each card. The cards are placed in a box. Alan removes two randomly chosen cards from the box. He then calculates the product of the numbers on the two cards. What is the average value (mean) of the result that he gets? Express your answer as a common fraction. 1. _____

2. Let $N = 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdots 163$. What is the largest positive integer n such that 3^n is a factor of N? 2. _____

3. A group of robbers stole a quantity of thin silk, and decided to share it equally. If each robber received 6 *p'i* of silk, there would be 6 *p'i* left over. If each robber got 7 *p'i*, then 7 *p'i* more silk would be needed than they stole. How many *p'i* of silk did each robber actually get? Express your answer as a common fraction. 3. _____ *p'i*

4. In the figure below, $ABCD$ is a parallelogram, DP is perpendicular to AB, and DQ is perpendicular to BC. Given that the lengths of DP, PB, BQ, and QD are 4, 7, 1, and 8 centimetres respectively, how many square centimetres are in the area of parallelogram $ABCD$? Express your answer as a common fraction.

4. _____ cm^2



From Face-off Stage

1. How many even numbers are there between -101 and 101?

1. _____

2. What is the value of $2007^2 - 2005^2$?

2. _____

3. On July 1, 2006, the sun rises at 5:11 AM and sets at 9:21 PM. At what time on July 1, 2006 is it exactly halfway between sunrise and sunset?

3. _____

4. The sum of 11 consecutive integers is 110. What is the largest of these 11 integers?

4. _____

5. A car travels 5 kilometres in 4 minutes. At this speed, how many seconds does it take to travel 1 kilometre?

5. _____

6. The figure below represents a square grid of 9 points in which every point is at unit distance from its nearest horizontal or vertical neighbours. How many lines are there that contain 2 or more points of the grid?

6. _____

7. Express

$$\frac{4.8 \times 10^{18}}{1.2 \times 10^{20}}$$

as a common fraction.

7. _____

8. Alan started with a number x. He added 10 to it, multiplied the result by 10, then subtracted 10, ending up with 200. What is the value of x?

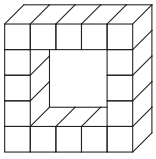
8. _____

9. Express $\sqrt{\frac{1}{25} + \frac{1}{144}}$ as a common fraction.

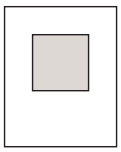
9. _____

10. The figure below was constructed by cementing together sixteen 1 cm by 1 cm by 1 cm cubes. What is the surface area of the figure, in cm^2?

10. _____



11. A rectangular poster is 40 cm wide. There is a 20 cm by 20 cm square picture on the poster. The picture takes up one-fifth of the area of the poster. How many cm are in the height of the poster? 11. _____



12. The average of five numbers is 80. The average of the first three of these numbers is 70. What is the average of the last two of the numbers? 12. _____

13. Alphonse rolls three standard dice once. What is the probability that the sum of the numbers rolled is equal to 4? Express your answer as a common fraction. 13. _____

14. What is the largest digit k such that the five-digit number that has decimal representation $88k88$ is a multiple of 12? 14. _____

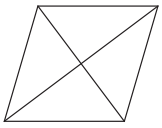
15. If $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{2}{3}$ and $x+y=8$, what is the value of xy? 15. _____

16. Suppose that for all n, 16. _____

$$f(n+2)=f(n)+f(n+1).$$

Given that $f(3)=8$ and $f(4)=5$, what is the value of $f(1)$?

17. What is the number of units in the perimeter of a rhombus whose diagonals have lengths 3 and 4 units? 17. _____



18. How many of the perfect squares between 1^2 and 100^2 have decimal representation with the units digit equal to 4? 18. _____

19. Four points A, B, C, D are on the same line as in the picture below. If 19. _____

$$\frac{AB}{BC}=\frac{1}{2} \quad \text{and} \quad \frac{BC}{CD}=\frac{4}{5},$$

what is the value of $\dfrac{AB}{BD}$? Express your answer as a common fraction.

20. What is the 40-th number in the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, ...? 20. _____