Regional 2006 Face-off Questions

1. What number is 50 less than 50% of 50?

2. If $\frac{6}{x} = \frac{1}{5}$, what is $6x$?

3. An ant starts at $(0, 0)$ and travels in a straight line towards $(19, -3)$. One-quarter of the way to $(19, -3)$, it takes a nap. What is the sum of the coordinates of the point where it naps?

4. Mall-Wart's normal profit on a dress is 200%. At the end of the season it holds a 50% off sale. What is Mall-Wart's percent profit on a dress when it is on sale?

5. A house has 5 basement windows and 3 doors to the outside. In how many different ways can a burglar enter the house through a basement window and leave through a door?

6. A circle is 1 metre in diameter. What is the number of square metres in the area of the circle? Express your answer in terms of π.

7. The price of 120 grams (net weight) of canned tuna is \$1.32. What is the cost, in dollars, per 100 grams of tuna?

8. Alice and Bob play a game in which ties are impossible. The probability that Bob wins is three-quarters of the probability that Alice wins. What is the probability that Alice wins?

9. The diameter of a circle is increased by 10%. By how many percent does the circumference increase?

10. If $x + 6y = 9$ and $x - 6y = -2$, what is the value of y?

11. Alan went to Mall-Wart and bought 2 pants and 3 shirts for a total of \$100. Each pant cost \$11.50 more than each shirt. How many dollars did each shirt cost?

12. How many different primes divide 50! but do not divide 25!?

13. The figure below consists of 11 points: the 10 vertices of a regular decagon and the center of the decagon. How many triangles have one vertex at the center of the decagon, and the other two at vertices of the decagon?

14. What is the units digit in the decimal expansion of $\dfrac{99!}{95!}$?

15. What is $\dfrac{3}{4}$ divided by $\dfrac{5}{6}$?

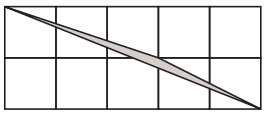
16. Alicia and Beti together drank a 750 ml bottle of wine. Alicia drank 50% more wine than Beti. How many ml of wine did Alicia drink?

17. The greatest common factor of 4807 and 4853 is not equal to 1. What is it?

18. The gas tank of Alicia's car has capacity 57 litres, and was one-third full when she went to the gas station. She filled up the rest of the tank. If gas cost \$1.02 a litre, how many dollars did the fill-up cost?

19. If n is a perfect square, how many possible values are there for the units digit in the decimal expansion of n?

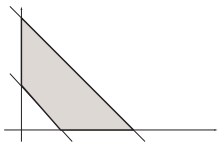
20. Each of the 10 small squares in the figure below has area 1 square unit. What is the number of square units in the area of the shaded triangle?



21. Alan puts \$12 under his mattress every month. How many years will it take for these savings to build up to \$3600?

22. If x is 60% of y, what percent is $3x$ of $2y$?

23. A trapezoidal region is bounded by the x-axis, the y-axis, and the lines $x + y = 4$ and $x + y = 10$. What is the number of square units in the area of the region?



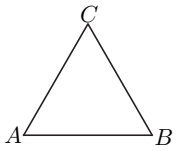
24. What is the positive difference between 2 and the square of $\frac{17}{12}$? Express your answer as a common fraction.

25. Four fair coins are tossed. What is the probability we get more than one head?

26. A high school sprinter ran 100 metres in 12 seconds. What was the sprinter's average speed in kilometres per hour?

27. A tub of Alicia's favourite ice cream costs \$4.80. What is the largest number of tubs she can buy if she has \$500?

28. How many squares have two of their vertices among the vertices of the equilateral triangle ABC?



29. Suppose that $a_1 = 2$ and $a_{n+1} = 2a_n - 1$ for all $n \geq 1$. What is the value of a_7?

30. A cat sleeps for 20 hours a day. It spends 60% of its waking hours doing nothing, 15% grooming itself, and 15% eating. The rest of the time it scratches the furniture. How many minutes a day does it scratch the furniture?

31. Three cubical dice each have the number 1 marked on three of the faces, and -1 on the other three. The three dice are tossed. What is the probability that the sum of the numbers on the 'top' faces is 1?

32. A 1 km jog uses up 70 calories. A small Dairy Queen chocolate malt has 525 calories. How many km must one jog to use up the calories in a small chocolate malt? Give the answer to the nearest tenth of a km.

33. What is the value of $(0.4)^2 - (0.4)^3$?

34. The NHL once had 6 teams. Each year, every team played 50 regular season games. How many regular season NHL games were played each year?

35. Imelda has 365 different pairs of shoes. If she picks two of the shoes at random, what is the probability that the shoes match?

36. Alan has 5 pennies and 10 dimes. In how many ways can he distribute these coins between his left pocket and his right pocket? Either pocket could end up empty. The pennies are identical, as are the dimes.

37. The square of the sum of two numbers is 100. The square of their difference is 99. What is the product of the two numbers?

38. There are 120 ways to arrange the letters of the word 'ANGLE' in a row. Suppose these 120 arrangements are listed alphabetically, from AEGLN to NLGEA. What is the 60-th arrangement in the list?

39. What is $0.\overline{540}$ (that is, $0.540540\ldots$)? Give the answer as a fraction in lowest terms.

40. A sequence has the property that for any three consecutive terms, the third term is equal to the sum of the previous two. If the 100-th term of the sequence is 1, and the 99-th term is 3, what is the 96-th term?

41. An operation $\star$ is defined by $x \star y = x^2 - y^2$. What is the value of $(4 \star 5) \star 6$?

42. Six fair coins are tossed. What is the probability that the positive difference between the number of heads and the number of tails is not equal to 1?

43. Of the students in Mr. Alpher's Math 9 class, 62.5% are in grade 9, 25% are in grade 8, and the remaining 6 students are in grade 10. How many students are in Mr. Alpher's Math 9 class?

44. What is the sum of the solutions of the equation
$3(x-4)^2 + 5 = 6$?

45. What is the remainder when $2003 + 2004 + 2005$ is divided by 2006?

46. Alan says to Beth: "I only have 65 dollars. Give me 10 dollars. If you do, you will still have twice as much money as I will have." How many dollars does Beth have?

47. An integer from 1 to 1000 (inclusive) is chosen at random. What is the probability that its cube root is greater than 5?

48. Let A have coordinates $(3, 0)$ and let B have coordinates $(0, 5)$. What is the number of square units in a square that has the segment AB as a diagonal?

49. If one-half of one-third of one-quarter of a number is 120, what is one-third of one-quarter of one-fifth of the number?

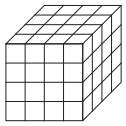
50. What is the sum of all integer values of x such that $(x + 1)^2 \leq 25$?

51. Given that n is an integer and $15n$ is a multiple of 21 and $21n$ is a multiple of 15, what is the largest integer that *must* be a factor of n?

52. What is the sum of all positive integer values of n such that the least common multiple of 12 and n is 24?

53. What is the value of $\dfrac{15^2 - 9^2}{13^2 - 5^2}$?

54. Alan has 32 red and 32 blue $1 \times 1 \times 1$ cubes. He glues all of these together to make a $4 \times 4 \times 4$ cube. What is the least possible number of red 1×1 faces on the outside of the $4 \times 4 \times 4$ cube?



55. If $\dfrac{2x + 1}{x} = \dfrac{5}{6}$, what is the value of x?

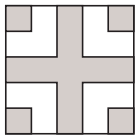
56. If 75% of a number is 60, what is 85% of that number?

57. What is the least common multiple of 18, 24, 32, and 36?

58. What time is it 1500 seconds after 1:20?

59. What is the positive difference between 90% of 125,000 and 90% of 25,000?

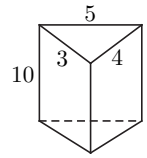
60. The sides of a square are divided into 5 equal parts to create the pattern below. What is the ratio of the area of the shaded part of the square to the area of the whole square? Express the answer as a common fraction.



61. What is the product of the odd integers from -5 to 5, inclusive?

62. What is the positive difference between $\frac{5}{6}$ and its reciprocal?

63. In the triangular glass prism below, the sides of the end triangles are 3, 4, and 5 cm. The prism has height 10 cm. What is the number of square cm in the total surface area of the prism?



64. What is the sum of the positive integers that are less than 12 and not a multiple of 3?

65. Let $f(x) = x^2 + x + 41$. What is $f(40) - f(-40)$?

66. What is $\frac{10!7!4!}{9!6!3!}$?

67. What is the product of the first five positive odd integers?

68. A cubic centimetre of gold has mass 19.32 grams. How many kilograms are in the mass of a 1 metre by 1 metre by 1 metre cube of gold?

69. On July 1, 2006, the sun rises at 5:11 AM and the sun sets at 9:21 PM. At what time on July 1, 2006 is it exactly halfway between sunrise and sunset?

70. Alicia's restaurant meal cost \$125. She decided to leave a 12% tip. What total number of dollars did she pay (bill plus tip)?