

Page 1: Problem Solving

1. A class of 30 students took a test, and the class average was 70. The five students who failed had marks of 20, 25, 25, 30, and 40. What was the average mark among the students who didn't fail? Give the answer correct to 1 decimal place.

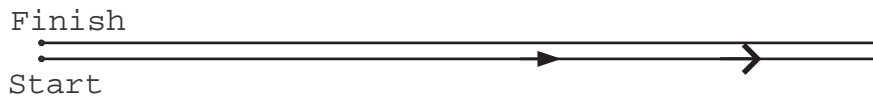
1.

2. Beta has twice as many pennies as Alpha. Gamma has three times as many pennies as Beta. Between the three of them, they have fewer than 80 pennies. What is the largest possible number of pennies that they could have between them?

2.

3. Xaviera jogs at a steady rate of 5 minutes per kilometre. Yolande jogs at 7 minutes per kilometre. They start at the same time on an out and back run that consists of 15 km due east followed by 15 km due west. How many kilometres from the finish line is Xaviera when they pass next to each other? Give the answer correct to 1 decimal place.

3.



4. It so happens that 7700625 is a perfect square, that is, the square of a whole number, and that 7706176 is the next perfect square. What is the first perfect square greater than 7706176?

4.

Page 2: Combinatorics

5. In a game, you toss two standard dice, a silver one and a gold one. Your *score* is the number showing on the silver one, plus twice the number showing on the gold one. What is the probability that your score is 8? Express your answer as a fraction in lowest terms.

5.

6. There are 5 people in a family, all of different heights. We want to line them up for a picture, with the tallest person in the middle, and so that as we go from left to right, the heights of the people increase and then decrease. How many ways are there to do this?

6.

7. Let A , B , and C be the measures, in degrees, of the angles of a triangle, where $A \leq B \leq C$. How many possibilities are there for the ordered triple (A, B, C) , given that each of A , B , and C is a multiple of 15?

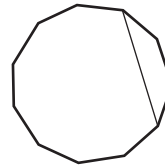
7.

8. How many 4-digit numbers are there which obey all three of the following rules: (i) no digit other than 1, 2, 3, or 4 is to be used; (ii) a digit may occur more than once; (iii) as you read the number from left to right, digits never decrease (so 1134 is OK, but 3314 is not)?

8.

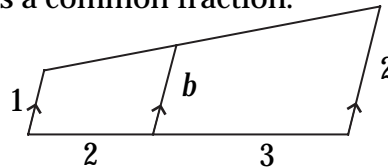
Page 3: Geometry

9. A regular polygon has 11 sides. How many diagonals does it have?
 A *diagonal* is a line segment that joins two corners but is not a side.
 One diagonal is shown in the picture.



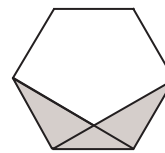
9.

10. Lines that look parallel in the picture *are* parallel. Express b , the length of the “middle” line segment, as a common fraction.



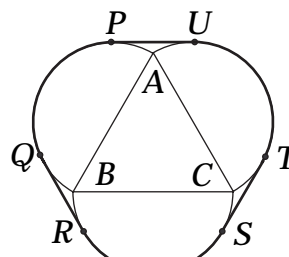
10.

11. The figure below is a regular hexagon with area 1. Express the area of the shaded region as a common fraction.



11.

12. Each side of triangle ABC has length 2. Three semicircles are constructed outside ABC with the three sides AB , BC , CA as diameters. A thread $PQRSTUP$ is tied tightly around the resulting “three-leaf clover” shape so that PQ , RS , TU are arcs of the semicircles with diameters AB , BC , CA respectively and QR , ST , UP are line segments. Find the area enclosed by the thread. Give your answer in terms of π .



12.