Blitz, Page 1

1. Simplify: $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{5}{6}}{\frac{5}{12}}$ 1. ____________

2. Approximately how many seconds are there in two-sevenths of a minute? Round your answer to the *nearest* second. 2. ____________ seconds

3. Alphonse has equal numbers of nickels, dimes, quarters, and loonies. If the total value of these coins is less than \$20.00, what is the maximum possible total value of these coins? Give the answer in dollars, to the nearest cent. 3. ____________ dollars

4. Three friends (call them A, B, and C) go to a movie theatre. There are 5 consecutive empty seats in the front row. All other seats are occupied. How many ways are there to seat the friends, if they need to occupy consecutive seats? Two ways are shown in the picture below. 4. ____________ ways

		B	C	A

	A	C	B	

5. A rectangle has area 1000 units^2. A new rectangle is constructed by increasing the length of the original rectangle by 10%, and decreasing the width by 10%. What is the area of the new rectangle? 5. ____________ units^2

6. What is the value of $2^5 - 2^4 + 2^3 - 2^2 + 2^1 - 1$? 6. ____________

7. A cubical die has its faces labelled with the numbers 1, 3, 5, 7, 9, and 11 instead of the usual 1 to 6. If two such dice are tossed, what is the probability that the sum of the numbers on the two "up" faces is 6? Express your answer as a common fraction. 7. ____________

8. Let $a_0 = 0$, and for any $n \geq 1$, let $a_n = n^2 - a_{n-1}$. What is the value of a_3? 8. ____________

9. What is the integer closest to $\sqrt{2009}$? 9. ____________

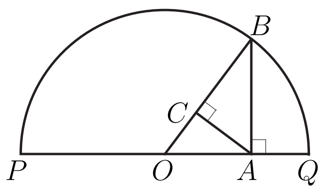
10. What is the smallest positive integer n such that 100 divides $n!$? 10. ____________

11. Suppose that x and y are positive integers such that $400x+9y = 2009$. What is the largest possible value of y? 11. ____________

12. A box of 6 doughnuts costs \$3, and a box of 13 doughnuts costs \$6. What is the least number of dollars Alphonse needs to spend in order to buy *exactly* 175 doughnuts? 12. ____________ dollars

13. After playing 20 games into the season, the Burnaby Bruisers had won 6 games and lost 14. After playing these 20 games, they fired the water boy. Over the rest of the season the Bruisers lost only 7 games and won the rest. Over the entire season, they won exactly two-thirds of the games they played. How many games did they play during the entire season? 13. ____________ games

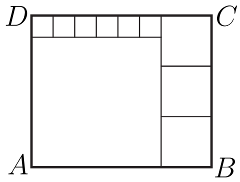
14. The figure below is a half-circle with centre O. Given that $PA = 13$ and $AQ = 3$, what is the length of OC? Express your answer as a common fraction. 14. ____________ units



15. The mean and the median of a collection of 5 different positive integers are both equal to 20. What is the largest possible integer in the collection?

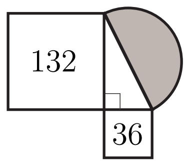
15. ____________

16. The rectangle $ABCD$ is divided into 10 squares as in the picture below. If the side of one of the smallest squares (say the one at the corner D) is 3 units, how many units are in the base AB?



16. ____________ units

17. Squares are erected on the two legs of a right-angled triangle. These squares have areas 36 and 132 as shown. A semicircle (shaded) is drawn with the hypotenuse as diameter. What is the area of the semicircle? Give your answer in terms of π.

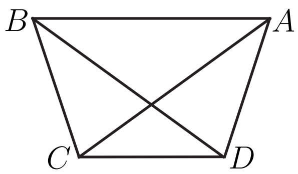


17. ____________ units^2

18. If $|x| + |2x + 3| \leq 60$, what is the largest possible value of $|x|$?

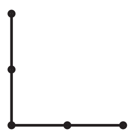
18. ____________

19. In quadrilateral $ABCD$, $AB = AC = BD$, and $BC = CD = DA$. What is the degree measure of $\angle ABC$?



19. ____________ degrees

20. We are given 5 points, which have coordinates $(0, 0)$, $(1, 0)$, $(2, 0)$, $(0, 1)$, and $(0, 2)$. How many ways are there to choose 3 of these points so that the 3 chosen points are the vertices of a triangle? Note that for example choosing $(2, 0)$, $(0, 1)$, and $(0, 2)$ is the same as choosing $(0, 1)$, $(2, 0)$, and $(0, 2)$.



20. ____________ ways

21. A large bottle contains 4 litres of a solution which is 5% acetic acid (and the rest water). How much of a solution which is 20% acetic acid should we add to the bottle to obtain a solution which is 7% acetic acid? Give your answer as a common fraction, in litres.

21. ____________ litres

22. What is the area of the triangular region enclosed by the 3 lines that have equations $x - y = 0$, $x + y = 2$, and $x = 10$?

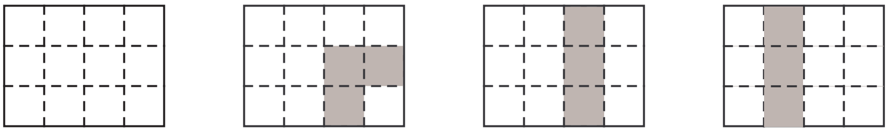
22. ____________ units^2

23. Let $N = 4^8 \times 5^9$. How many digits are there in the decimal representation of N?

23. ____________ digits

24. The rectangle on the left represents a 4×3 sheet of stamps, 12 stamps altogether. How many different ways are there to choose a set of 3 stamps which are *connected*? Connection must be through shared edges: a shared vertex is not good enough. The three pictures on the right show three different ways of doing the job.

24. ____________ ways

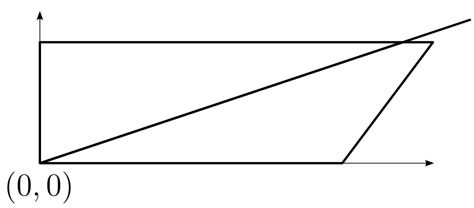


25. Five (5) cards with the number 1 written on them, and four (4) cards with the number 2 written on them, are placed in a box. You randomly select 3 of these 9 cards. What is the probability that the sum of the numbers written on the 3 selected cards is odd? Express your answer as a common fraction.

25. ____________

26. The vertices of a trapezoid are $(0,0)$, $(10,0)$, $(10+m,4)$, and $(0,4)$. The line $y = x/m$ divides the trapezoid into two polygons of equal area. What is the value of m? Express the answer as a common fraction.

26. ____________



Bull's-eye, Page 1: Problem Solving

1. A store sells only bicycles (2 wheels each) and tricycles (3 wheels each). The store has exactly as many bicycles as tricycles. Given that the bicyles and tricycles in the store have a combined total of 330 wheels, how many tricycles are in the store?

   1. ____________ tricycles

2. SellHigh™ bought apples from a farmer, at 12 apples for $1. SellHigh then sold all the apples in its Vancouver store at 2 apples for $1. SellHigh's total profit on the apples was $3000. How many dollars did SellHigh pay the farmer for the apples?

   2. ____________ dollars

3. A box-shaped pool is 25 metres long, 12 metres wide, uniformly 1 metre deep, and full of water. Water is leaking from the pool at 1000 cubic centimetres per minute. How many minutes will it take for the water level in the pool to go down by 1 centimetre?

   3. ____________ minutes

4. There are two candles, one short and thick, the other tall and thin. They burn at different rates. The short thick candle can burn for 120 minutes. Both candles were lit at the same time, and after 30 minutes they were both the same height. After 30 additional minutes, the (originally) tall candle was half the height of the (originally) short candle. What is the total expected burn time, in minutes, of the (originally) tall candle?

   4. ____________ minutes

Bull's-eye, Page 2: Combinatorics and Numbers

5. What is the sum of all the positive integers that divide 60? (Note that 1 and 60 divide 60.)

5. ____________

6. Four people (A, B, C, and D) line up in a row at random. What is the probability that A and B are next to each other but C and D are not next to each other? Express your answer as a common fraction.

6. ____________

7. The sum of four *different* positive integers is equal to 300. If S is the smallest of the four positive integers, and B is the biggest, what is the smallest possible value of $S + B$?

7. ____________

8. Twenty (20) people come to a party. We know that 11 of the people are friends with everyone else who came to the party. Also, the other 9 people each have exactly 13 friends at the party. (Assume that if A is a friend of B, then B is a friend of A. Assume also that A is never a friend of A.)

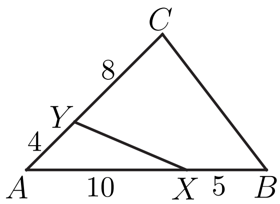
   Each person shakes hands with each of his/her friends. What is total number of handshakes?

8. ____________ handshakes

Bull's-eye, Page 3: Geometry

9. How many lines of symmetry does a regular hexagon have? 9. ____________ lines

10. In the diagram below, $AX = 10$, $XB = 5$, $AY = 4$, and $YC = 8$. What is the ratio of the area of $\triangle AXY$ to the area of $\triangle ABC$? Express your answer as a common fraction. 10. ____________



11. The *slant* height of a cone is 41, and the ordinary height (distance from the vertex to the centre of the base) is 40. If the volume of the cone is $N\pi$, what is the value of N? 11. ____________

12. A ball of ice of radius 5 cm is placed in a tall empty cylindrical glass with the same radius. When the ice melts, every 11 cm^3 of ice turns into 10 cm^3 of liquid. When all the ice has melted, what is the height, in cm, of liquid in the glass? Express the answer as a common fraction. 12. ____________ cm

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. Let N be the smallest positive integer such that each of N and $N+1$ has exactly 4 positive divisors. What is the value of N?

1. ____________

2. Let a be the number of divisors of $6!$, and let b be the number of divisors of $7!$. What is the value of $\frac{a}{b}$? Express your answer as a common fraction.

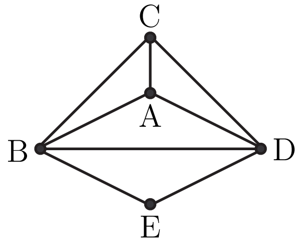
2. ____________

3. A cube is inscribed in a sphere of radius 3 cm. (So the cube is *inside* the sphere, and the all the corners of the cube touch the boundary of the sphere.) What is the surface area of the cube?

3. ____________ cm^2

4. The black circles labelled A, B, C, D, and E represent cities, and the straight lines are highways between them. We want to travel from city to city in such a way that we travel on every road exactly once. (We may go through a city more than once.) It turns out that every path that works begins in a certain city X, and ends in a certain city Y, or vice-versa. What are these two cities? For example, if the path must begin at C and end at D, or vice-versa, your answer should be CD (or if you like, DC).

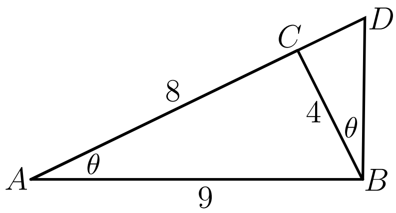
4. ____________



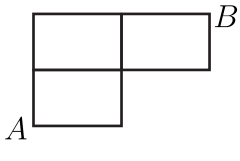
Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. What is the largest integer which is less than $\sqrt[3]{4100} \times \sqrt[4]{4100}$? 5. ____________

6. Triangle ABC has $AB = 9$, $AC = 8$, and $BC = 4$. Line segment AC is extended to D in such a way that $\angle CBD = \angle CAB$. What is the length of the line segment CD? Express your answer as a common fraction. 6. ____________ units



7. The lines in the diagram below represent the streets of a village. How many ways are there to drive from A to B, using village streets, if one is not allowed to travel along any block (segment) more than once? (One can pass through an *intersection* more than once.) 7. ____________ ways



Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

8. For any positive integer n, let $S(n)$ be the sum of the (decimal) digits of n. For example, $S(8) = 8$ and $S(47) = 11$. How many two-digit numbers n are there such that $S(S(n)) = 5$?

8. ____________ numbers

9. Consider the angle between the hour hand and the minute hand of a watch. There are times when the angle between these hands is exactly 180 degrees (example: 6:00 o'clock). Find the sum of all these times, in the period from 1:00 pm to 4:00 pm the same day. Give the answer as a common fraction, in hours.

9. ____________ hours

10. A class of 12 students is currently divided into 4 working groups of 3 students each, namely {A,B,C}, {D,E,F}, {G,H,I}, and {J,K,L}.

Suppose that you want to regroup these 12 students into 4 groups of 3 students each, so that no 2 students who are currently in the same group will end up in the same group. In how many ways can this be done? An example of a valid way is {A,E,H}, {B,F,J}, {C,G,K}, and {D,I,L}. An example of an invalid way of regrouping is {A,E,F}, {B,G,J}, {C,H,K}, and {D,I,L}, because E and F are currently in the same group, so must end up in different groups.

10. ____________ ways