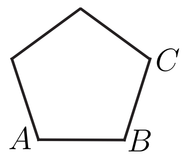
Blitz, Page 1

1. Alicia tosses 3 fair coins. What is the probability that she gets at least 1 head? Express your answer as a common fraction. — 1. ____________

2. It took Anita 2.25 hours to walk 12.5 km. At this rate, how many hours will it take for her to walk a total of 50 km? — 2. ____________ hours

3. What is the value of $0.15 + 0.2 + 0.25 + 0.3 + 0.35$? Express your answer as a decimal. — 3. ____________

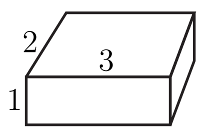
4. The points A, B, and C in the figure below are 3 consecutive vertices of a regular pentagon. What is the degree measure of angle ABC? — 4. ____________ degrees



5. After getting all his cavities filled by the dentist, Andy celebrated by buying candies which were on sale at 32 per dollar. He spent \$3.50 on the candies. How many candies did he buy? — 5. ____________ candies

6. Simplify: $\dfrac{4+5+6+7}{1+\frac{1}{2}+\frac{1}{3}}$. — 6. ____________

7. What is the total (outer) surface area of a closed box with sides 1, 2, and 3? — 7. ____________ units^2



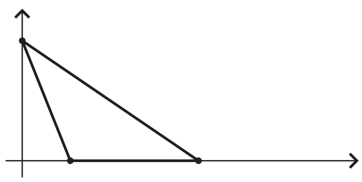
8. Let $x = 1.23 + 4.56 + 7.89$. What is the integer nearest to x? 8. ____________

9. Express $\dfrac{(99!)(101!)}{(98!)(102!)}$ as a common fraction. 9. ____________

10. Alfie is running at a speed of 5 metres per second. What is his speed in kilometres per hour? 10. ____________ km/hr

11. At Big Box Secondary School, each teacher teaches 4 classes per day, each student takes 5 classes per day, each class has 35 students, and there are 150 teachers. How many students are there at the school? 11. ____________ students

12. The vertices of a triangle in the coordinate plane are at $(1, 0)$, $(x, 0)$, and $(0, x-1)$. Given that $x > 1$, and that the triangle has area 18, what is the value of x? (The picture is not drawn to scale.) 12. ____________



13. In a "mixed" chess tournament, every person played 5 games, 3 of them against a male player, and 2 against a female player. Altogether, there were 300 games. How many males played in the tournament? 13. ____________ males

14. Suppose that $|x + 1| + |x - 1| \geq 10$ and $2x + |x - 1| \geq 6$. What is the smallest possible value of x? 14. ____________

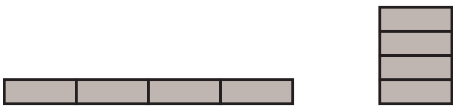
15. King Charles the Short-Sighted rode all the way around the boundary of his kingdom, which is a circle with diameter 100 kilometres. While he rode along, he could see only 20 metres in any direction. What is the total area (both inside and outside his kingdom) of the region that he saw? Assume the world is flat. Express your answer in terms of π.

15. ____________ km^2

16. What is the greatest prime factor of $13^{12} + 13^{13} + 13^{14}$?

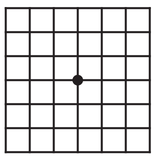
16. ____________

17. We have 4 identical rectangular tables, each of which has short side equal to a, and long side equal to b. When the tables are pushed together short sides to short sides (left part of picture), the resulting long table has perimeter which is 2 times the perimeter when the tables are pushed together long sides to long sides (right part of picture). What is the ratio of a to b? Express your answer as a common fraction.



17. ____________

18. The lines below represent the streets of a small town. All blocks in the town are squares. Alphonse is standing at the street intersection in the middle of town indicated by the small black circle. How many intersections can he end up at by walking for 3 or fewer blocks? Don't forget about the intersection he starts at, which he can reach by walking for 0 blocks, or 1 block north and 1 block south.



18. ____________

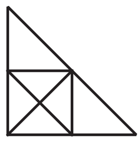
19. A car uses 12 litres of gas to travel 100 km. If gas costs \$1.50 a litre, how much does it cost to buy the gas to travel 12 km? Give your answer in dollars, to the nearest cent.

19. ____________ dollars

20. Let $f(x, y, z) = \dfrac{xyz}{x+y+z}$. What is the value of $f(3\sqrt{7},\ 2\sqrt{7},\ \sqrt{7})$?

20. ____________

21. How many triangles are there altogether in the figure below? 21. ____________ triangles

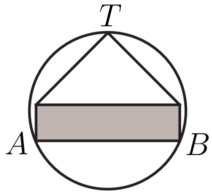


22. What is the largest integer n such that $4n+1$ is a multiple of $n+4$? 22. ____________

23. In how many different ways can 6 people (A, B, C, D, E, and F) be lined up in a row so that A and B are next to each other, C and D are next to each other, and E and F are *not* next to each other? 23. ____________ ways

24. What the largest integer n such that $\dfrac{1440}{n^2-1}$ is an integer? 24. ____________

25. The shaded rectangle below has base AB of length 4, and has height 1. An isosceles triangle is erected with base the side opposite to AB. The triangle is right-angled at T. A circle is drawn passing through A, B, and T. What is the radius of the circle? Express your answer as a common fraction. 25. ____________ units



26. There is an integer a such that $a^{11} = 177917621779460413$. (Note that a^{11} is an 18-digit number.) What is the value of a? 26. ____________

Bull's-eye, Page 1: Problem Solving

1. The average of x and $2y$ is 55. The average of x and $2z$ is 59. What is the average of x, y, and z?

   1. ____________

2. Alphonse works part-time. He earns \$6 per hour on weekdays (Monday to Friday) and \$8 per hour on weekends. This week he worked for a total of 32 hours, and earned \$220. How many weekday hours did Alphonse work?

   2. ____________ hours

3. Alicia weighed 168 pounds when she went on a diet, and her body was 30% fat by weight. On her diet, she lost fat only. After a year of dieting, her body was 20% fat by weight. How many pounds did she weigh then?

   3. ____________ pounds

4. Let a and b be real numbers such that

   $$a + \frac{1}{b} = 9 \qquad \text{and} \qquad b + \frac{1}{a} = 10.$$

   What is the value of $ab + \dfrac{1}{ab}$?

   4. ____________

Bull's-eye, Page 2: Combinatorics and Numbers

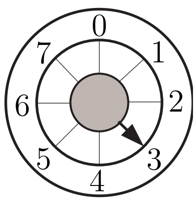
5. Two fair dice are tossed. What is the probability that the *product* of the two numbers so obtained is divisible by 3? Express your answer as a common fraction. 5. ____________

6. Find the positive difference between the largest and the smallest of the fractions 6. ____________

$$\frac{3}{4}, \qquad \frac{4}{5}, \qquad \frac{11}{15}, \qquad \frac{23}{30}, \qquad \frac{47}{60}.$$

Express your answer as a common fraction.

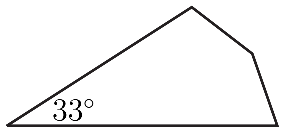
7. A combination for a simple bicycle lock is a sequence (a, b, c), where a, b, and c can be any of 0, 1, 2, 3, 4, 5, 6, or 7, but *adjacent* numbers in the combination are different. For example $(5, 0, 7)$ is a legitimate combination, as is $(5, 0, 5)$, but $(2, 4, 4)$ is not allowed. How many combinations (combos) are possible, altogether? 7. ____________ combos



8. Suppose that x and y are *positive* integers such that $xy = x+y+2009$. What is the smallest possible value of $x + y$? 8. ____________

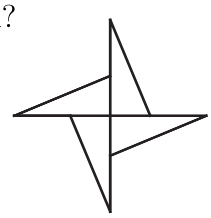
Bull's-eye, Page 3: Geometry

9. The measures of the four interior angles of a 4-sided convex polygon form an arithmetic progression. The smallest angle has degree measure 33°. What is the degree measure of the second smallest angle?



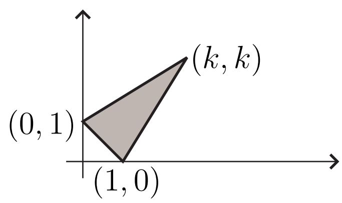
9. ____________ degrees

10. The pinwheel below is made by putting together four right-angled triangles whose legs are $\sqrt{7}-1$ cm and $\sqrt{7}+1$ cm in length. What is the number of cm in the perimeter of the pinwheel?



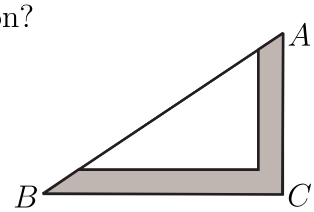
10. ____________ cm

11. The vertices of the shaded triangle below have coordinates $(0,1)$, $(1,0)$, and (k,k) where k is positive. The triangle has area $\frac{8}{9}$ units^2. What is the value of k, as a common fraction?



11. ____________

12. In the picture below, which is not drawn to scale, $\triangle ABC$ is right-angled at C. The two legs AC and BC have length 40 and 60. The shaded region consists of all points *inside* $\triangle ABC$ which are at a distance less than or equal to 6 from one (or both) of the two legs of $\triangle ABC$. What is the area of the shaded region?



12. ____________ units^2

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. What is the probability that a randomly selected positive divisor of 2^{100} is a divisor of 2^{50}? Note that for any positive integer N, both 1 and N are divisors of N. Express your answer as a common fraction.

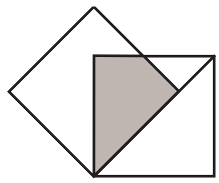
1. ____________

2. How many of the integers in the interval from 1 to 40 have exactly 4 (positive) divisors?

2. ____________ integers

3. Two 1×1 squares partially overlap as shown. What is the area of the region of overlap? Express your answer in simplest radical form.

3. ____________ units^2



4. How many integers between 100 and 999 are there whose "hundreds" digit is equal to the sum of the other two digits?

4. ____________ numbers

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. How many positive integers are there none of whose digits is 0 and the sum of whose digits is 5?

5. ____________ integers

6. In the year k, where $1 \leq k \leq 4$, the village of Ratland had $P(k)$ people, $D(k)$ dogs, $C(k)$ cats, and $R(k)$ rats. Suppose that:
(a) For $1 \leq k \leq 4$, $P(k) = D(k) + C(k)$ (the number of people is equal to the number of dogs plus the number of cats);
(b) For $1 \leq k \leq 3$, $P(k+1) = P(k) + 10$, $D(k+1) = D(k) + 15$, and $R(k+1) = R(k) - 100\,C(k) + 10000$;
(c) $R(1) = 10000$, $P(2) = 100$, and $D(4) = 75$.
What was the value of $R(4)$, the rat population in year 4?

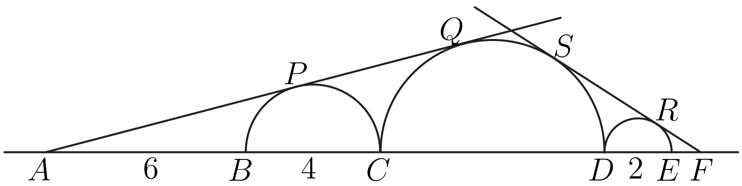
6. ____________ rats

7. When Mr. Lucky starts betting, he has 3 dollars. On any bet, he wins with probability 1/3 and loses with probability 2/3. If he wins a bet, the total amount of money he has triples. If he loses a bet, he loses 2/3 of the total amount of money he has. Mr. Lucky's objective is to walk away with 27 dollars in his pocket, and he can keep playing as long as he has at least 3 dollars. What is the probability that he reaches his objective? Express your answer as a common fraction.

7. ____________

8. The numbers in the set $\{1, 2, 3, 4, 5, 6, 7\}$ are divided into two groups. Let a be the product of the numbers in one group, and let b be the product of the numbers in the other group. What is the smallest possible value of $a + b$? 8. ____________

9. Three semicircles have diameters on the same line. The points A, B, C, D, E, and F are on the line, in that order. The leftmost semicircle has diameter $BC = 4$, the rightmost semicircle has diameter $DE = 2$. The middle semicircle has diameter $CD > 4$. A common tangent line to the leftmost two semicircles meets the line of diameters at A, and a tangent line to the rightmost two semicircles meets the line of diameters at F. Given that $AB = 6$, what is the length of EF? Express your answer as a common fraction. 9. ____________



10. A 2×2 sheet of paper is folded once to make a 2×1 rectangle. The rectangle is then folded once to make a 1×1 square package. Finally, this package is cut all the way through along the two diagonals. How many separate pieces has the original sheet of paper been cut into? 10. ____________ pieces