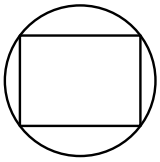
Blitz, Page 1

1. One-third of 105 is the same as seven-sixths of what number? 1. _____

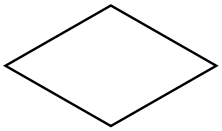
2. A rectangle has length 16 and width 12. What is the radius of the circle that passes through the four vertices of the rectangle? 2. _____ units



3. Which whole number is closest to 125% of 25? 3. _____

4. Express $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ as a fraction in lowest terms. 4. _____

5. What is the area of a rhombus with diagonals of length 4 and 7? 5. _____ units^2



6. Simplify $\dfrac{1}{1+\frac{2}{3}} - \dfrac{1}{1+\frac{3}{2}}$ 6. _____

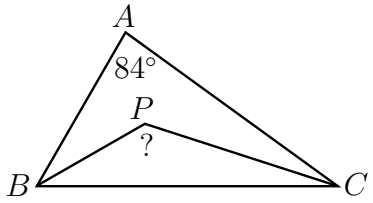
7. Let $F(x) = x^4+2x^3+3x^2+2x+1$. What is the value of $F(2)-F(-2)$? 7. _____

8. Express $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}$ as a common fraction. 8. _____

9. How many of the numbers from 1 to 100 can be written in the form $2^a 3^b$ where a and b are *positive* integers? 9. _____

10. One cell starts to divide at time $t = 0$. Any cell divides into 2 cells every minute, and the culture plate is full at $t = 20$ minutes. If we start with 4 cells starting to divide at $t = 0$ instead, how many minutes does it take until the plate is full? 10. _____ minutes

11. In $\triangle ABC$, the angle at A is $84°$. The bisectors of the angles at B and C meet at P. How many degrees are in the measure of $\angle BPC$? 11. _____ degrees



12. What is the greatest possible value of $p + q$ if $pq < 100$ and p and q are odd primes? 12. _____

13. A gambler went to the casino with 1 dollar. She made a series of six 1-dollar bets, winning or losing 1 dollar each time. She ended up with 1 dollar. In how many different orders (of winning/losing) could this have happened? Note that if you have no money you cannot bet. 13. _____ orders

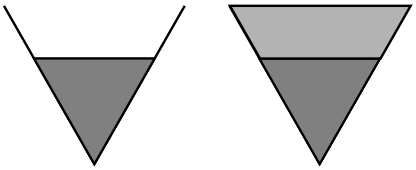
14. A box contains 3 black beads, 3 blue beads, 6 red beads, and 8 yellow beads. If you are blindfolded, how many beads must you take out in order to be sure of taking out at least 2 beads of each colour? 14. _____ beads

15. What is the value of $17^3 - 17^2 \cdot 16 - 17 \cdot 16^2 + 16^3$? 15. _____

16. A line passes through the points $(-1, 10)$, $(10, -1)$, and $(x, -10)$. What is the value of x? 16. _____

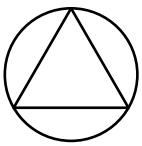
17. For how many positive integers n is the positive difference between $\sqrt{n}$ and 9 less than 1? 17. _____

18. Vinegar is poured into a conical cup of height 3 inches until the vinegar is 2 inches deep at its deepest point (please see the left-hand diagram). Then olive oil is poured into the cup until the cup is full (right-hand diagram). After the oil has been poured in, what common fraction of the cup's contents is oil? 18. _____

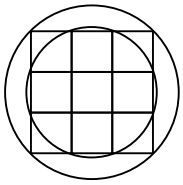


19. Alfred wrote down all the numbers from 111 to 999. How many times did he write the digit 0? 19. _____ times

20. An equilateral triangle with each side 6 cm is inscribed in a circle. If the area of the circle is $k\pi$ cm^2, what is the value of k? 20. _____



21. The large square is made up of nine 2×2 squares. Two circles whose center is the center of the large square pass through vertices of the small squares, as shown. What is the ratio of the area of the smaller circle to the area of the larger circle? Express the answer as a common fraction.



21. _____

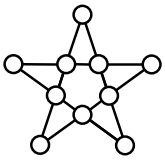
22. Given that $\left(x^2 + \dfrac{1}{x^2}\right)^2 = 100$, what is the value of $\left(x^2 - \dfrac{1}{x^2}\right)^2$?

22. _____

23. What is the smallest sum of money that can't be made up using 10 or fewer coins? Allowed coins are 1 cent, 5 cents, 10 cents, 25 cents, 1 dollar, and 2 dollars. Give the answer as a decimal, in dollars (an answer of 4.00 or 4.56 has the right shape).

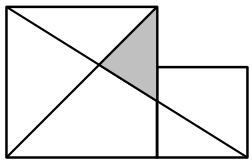
23. _____ dollars

24. It so happens that you can put the numbers 1, 2, 3, 4, 5, 6, 8, 9, 10, and 12 (every number from 1 to 12 except 7 and 11) in the circles below, one to each circle, so that the sums of the numbers in any 4 circles whose centres lie on the same line are all equal. What must each of these sums be?



24. _____

25. The larger square in the diagram has side 5 and the smaller square has side 3. What is the area of the shaded triangle? Express the answer as a common fraction.



25. _____ units^2

26. The number 130 has 8 positive factors, namely 1, 2, 5, 10, 13, 26, 65, and 130. How many positive integers *smaller* than 130 also have 8 positive factors?

26. _____ integers

Bull's-eye, Page 1: Problem Solving

1. Alan answered 60 questions on a quiz. He had 40% more right answers than wrong answers. How many questions did Alan get right?

1. _____ questions

2. The time in St. John's, Newfoundland, is 4.5 hours ahead of the time in Vancouver. A cargo plane left St. John's at 4:00 AM and landed in Vancouver at 9:45 AM the same day. The plane left Vancouver for St. John's 45 minutes later. The flight back to St. John's was shorter by 105 minutes than the flight to Vancouver. The plane arrived at St. John's in the evening. At what time did it arrive? Express the answer in Hours:Minutes format, using a 12-hour clock. Thus 10:40 has the right shape, and 22:40 does not.

2. _____ PM

3. Dan decided to distribute \$97 between his 5 kids, A, B, C, D, and E, giving each an integer number of dollars. He gave 1/2 (rounding up to the next \$) to A, 1/4 (rounding up to the next \$) to B, 1/5 (rounding up to the next \$) to C, 1/40 (rounding up to the next \$) to D, and the balance to E. How many dollars did E receive?

3. _____ dollars

4. We can put digits in the 5 squares below, one digit in each square, in a way that makes the statement below true. What 2-digit positive integer should be in the leftmost two squares?

4. _____

☐☐ percent of ☐☐☐ is 777

Bull's-eye, Page 2: Numbers and Combinatorics

5. How many ordered triples (x, y, z) of positive integers are there such that $x < y < z$ and $x + y + z = 12$?

5. _____ triples

6. Let $N = 1! + 2! + 4! + 8! + 16! + 32!$. What is the remainder when N^2 is divided by 16?

6. _____

7. How many ordered triples (a, b, c) are there such that a, b, and c are positive integers and $(a^b)^c = 64$? One such triple is $(64, 1, 1)$.

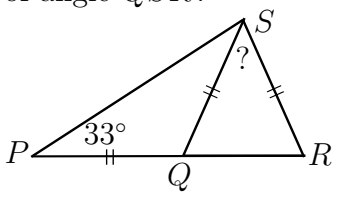
7. _____ triples

8. Five students will work on problem-solving in groups. Any group can consist of 1 to 5 students and each student must belong to exactly one group. In how many ways can the 5 students be divided into groups?

8. _____ ways

Bull's-eye, Page 3: Geometry

9. In the diagram, $PQ = SQ = SR$ and the degree measure of angle SPQ is 33°. What is the degree measure of angle QSR?

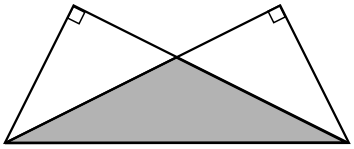


9. _____ degrees

10. Seven cubes are stacked one on top of the other. The volume of any cube is 10% more than the volume of the cube just above it. By how many percent is the side length of the bottom cube greater than the side length of the top cube?

10. _____ percent

11. The horizontal line segment at the bottom is the hypotenuse of two congruent right triangles. The legs of these triangles have lengths 1 and 2. What is the area of the shaded region? Express the answer as a common fraction.



11. _____ units^2

12. The base of a right circular cylinder has diameter 6, and the height of the cylinder is 8. The cylinder is enclosed in a sphere which is just large enough to contain the cylinder. What common fraction of the volume of the sphere is taken up by the cylinder?

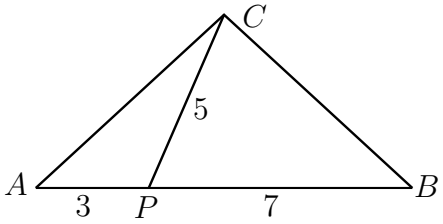
12. _____

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. Call an integer n *good* if the product of the positive integers that divide n is n^2. How many good integers are there in the interval from 1 to 27 (inclusive)? Please note that 1 is good, while 2 and 4 are not.

1. _____

2. Triangle ABC is isosceles, with $CA = CB$. Point P is on AB. Given that $AP = 3$, $BP = 7$, and $CP = 5$, what is the area of $\triangle ABC$? Express the answer as a decimal, rounded to the nearest tenth of a unit^2.



2. _____ units^2

3. A positive integer is called a *palindrome* if it doesn't change when the order of the digits is reversed (examples: 444 and 464). How many three-digit positive integers n are there such that n and $2n$ are both palindromes?

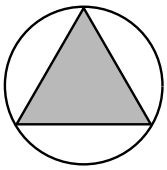
3. _____ integers

4. How many ordered pairs (a, b) are there such that a and b are integers (not necessarily positive) and $|a| + |b| \leq 3$?

4. _____ pairs

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. An equilateral triangle with area 1 cm^2 is inscribed in a circle. What is the number of cm^2 in the area of the circle? Express the answer as a decimal, rounded to the nearest one-hundredth of a cm^2. Note that π is approximately 3.14159.

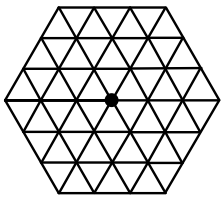


5. _____ cm^2

6. What is the probability that the sum of two distinct randomly chosen positive factors of 420 is odd? Express the answer as a common fraction. Note that 1 and 420 are factors of 420.

6. _____

7. A hexagonal island is pictured below. The small triangles are all equilateral with side 1 km. The lines are the roads of the island. In how many ways can Alicia start at the center of the island and walk to the sea along island roads, walking a total distance of 3 km?



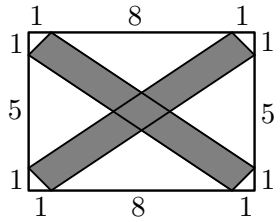
7. _____ ways

Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

8. Boiling water, at 100 degrees Celsius, is to be cooled to below 5 degrees. The water cools by 10 degrees in the first minute (so it is 90 degrees at time $t = 1$ minute). The water cools by 10×0.9 degrees in the second minute, by $10 \times (0.9)^2$ degrees in the third minute, by $10 \times (0.9)^3$ degrees in the fourth minute, and so on. At what *integer* number of minutes will the water first be below 5 degrees?

8. _____ minutes

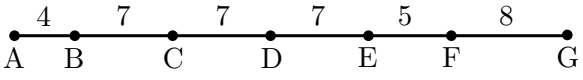
9. The rectangle below has length 10 cm and width 7 cm. An X-shaped figure (shaded) is drawn, with dimensions as shown. What is the area of the shaded figure, in cm^2? Express the answer as a common fraction.



9. _____ cm^2

10. Starting at 6:00 AM, and then every 4 minutes, trains leave the end-stations A and G of the AG Train route and go all the way to the other end-stations G and A. All trains stop for 1 minute at stations B, C, D, E, and F. Distances between stations, in km, are shown on the diagram. Note that the distance from A to G is 38 km. The average train speed between consecutive stations is 1 km per minute.

    Luciano got on a train that left Station A at exactly 9:00 AM, and got off a train at Station A before 11:00 AM the same day. On his trip he visited Station G. At any of the stations, Luciano had the choice to stay on the train, or get off the train and travel on the next train going in the opposite direction. Luciano maximized the distance he travelled. How many km did he travel in total?



10. _____ km