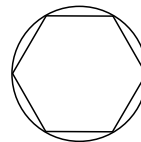Blitz, Page 1

1. Express the reciprocal of 0.55 as a common fraction.

1. _____

2. What is the smallest integer larger than $\sqrt{2012}$?
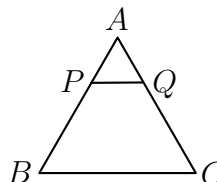
2. _____

3. Each edge of a regular hexagon has length $\dfrac{4}{\sqrt{\pi}}$. The hexagon is inscribed in a circle. What is the area of the circle, in square units?

3. _____ units^2

4. Alicia bought 45 litres of gasoline for \$54. If the price of gasoline goes up by 25%, how many litres of gasoline can Alicia buy for \$54?

4. _____ litres

5. Simplify: $\left(1 + \dfrac{1}{4}\right)\left(1 + \dfrac{1}{5}\right)\left(1 + \dfrac{1}{6}\right)\left(1 + \dfrac{1}{7}\right)$

5. _____

6. The area of equilateral triangle ABC is nine times the area of equilateral triangle APQ. What is the ratio of the perimeter of the trapezoid $PBCQ$ to the perimeter of $\triangle ABC$? Express the answer as a common fraction.

6. _____

7. Let $x \circledast y = x^2 - 2y^2$. What is the value of $3 \circledast (2 \circledast 1)$?

7. _____

8. Suppose that a and b are integers and $2^a - 2^b = 16$. What is the value of $a + b$?
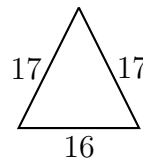
8. _____

9. A prism has 12 edges. How many faces does it have? Recall that a prism is a polyhedron for which there is a face of the polyhedron such that when the polyhedron is placed on the floor with that face down, then all horizontal cross-sections are the same.

9. _____ faces

10. Ali has 50% more money than Beth, who has 50% more money than Cecil. All together, they have \$950. How many dollars does Ali have?

10. _____ dollars

11. Simplify: $\sqrt{\sqrt{8}\sqrt{16}\sqrt{32}}$

11. _____

12. At the university, 30% of the students have a car, and 80% of the students who don't have a car have a bike. How many percent of the students have neither a car nor a bike?

12. _____ percent

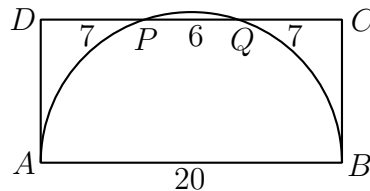13. What is the area of the triangle whose sides are 17, 17, and 16?

13. _____ units^2

14. What is the sum of the first 2012 terms of the following arithmetic sequence?
$$-1005, -1004, -1003, -1002, \ldots$$

14. _____

15. Evaluate: $\dfrac{10!\,7!\,4!}{9!\,6!\,3!}$

15. _____

16. The median of a list of 11 positive integers (not necessarily distinct) is 20 and their mean is 25. What is the largest possible integer in the list?

16. _____

17. Rectangle $ABCD$ has base 20. A semicircle is drawn that has the base AB as a diameter. This semicircle meets side CD in the points P and Q, where $DP = CQ = 7$ and $PQ = 6$. What is the height of the rectangle (that is, what is the length of line segment BC)? Express the answer in simplest radical form.

17. _____ units

D — 7 — P — 6 — Q — 7 — C

A — 20 — B

18. Six 5 dollar bills are placed in a row. Then every second bill is replaced by a 10 dollar bill. Then every third bill is replaced by a 20 dollar bill. After all the replacements are done, how many dollars in total are there in the row?

18. _____ dollars

19. A combined total of 2012 students participated in the last 8 Provincial Math Challengers competitions. The yearly participation numbers form an arithmetic sequence with a yearly increment of 3. What was the largest number of yearly participants during this period?
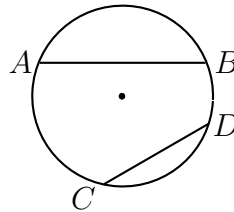
19. _____ students

20. In how many ways can 5 identical loonies be split between Aleph, Beth, and Gimel so that each of them gets at least 1 loonie? Only the totals that each person gets matter. For example, "Aleph is given 1 loonie, then Beth is given 1, then Alan is given 1, then Beth is given 1, then Gimel is given 1" is the same as "Beth is given 2, then Gimel is given 1, then Aleph is given 2."

20. _____ ways

Blitz, Page 4

21. Let $x = 2^{2012} + 3^{2012}$. What is the units digit of x?

21. _____

22. The integers i, j and k are even, and the integers l, m, and n are odd. Suppose that $0 < i < j < k < l < m < n$ and $\frac{i}{j} < \frac{k}{l} < \frac{m}{n}$. What is the smallest possible value of n?

22. _____

23. What is the smallest positive integer N such that N times 5! is a perfect cube?

23. _____

24. In the circle below, chord AB has length 22, and chord CD has length 16. Chord CD is twice as far from the centre of the circle as chord AB. What is the *square* of the radius of the circle?

24. _____ units^2

25. You toss 2 dice and record the sum. Then you do it again. What is the probability that the recorded sums are the same? Express the answer as a common fraction.

25. _____

26. A triangle has sides 3, 5, and 7. What is the *square* of its smallest height? Express the answer as a common fraction.

26. _____ units^2
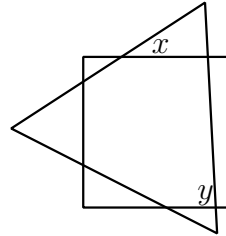
Bull's-eye, Page 1: Problem Solving

1. Alfie gave B one-half of the loonies Alfie had, and then 7 more. Alfie then gave C one-half of the loonies he had left, and then 7 more. After that, Alfie had no loonies left. How many loonies did Alfie start out with?

   1. _____ loonies

2. At the "Home Sweet Home" senior facility, the average age of the male residents is 70 years, the average age of the female residents is 75 years, and the average age of all residents is 73.5 years. What is the ratio of male residents to female residents of Home Sweet Home? Express the answer as a common fraction.

   2. _____

3. Dean and Dina each run exactly 600 m. They start at the same time and finish at the same time. Dina runs at a constant speed of 3 m/s, while Dan increases his speed at a constant rate for the first 300 m, and then decreases his speed by the same rate during the last 300m. What is the fastest speed (in m/s) that Dan reaches during the race?

   3. _____ m/s

4. You can use three different taps, alone or in combination, to fill a pool. If you use taps B and C only, it will take 9 hours to fill the pool. If you use all three taps (A, B, and C), it takes 7 hours. Tap B can fill the pool on its own in half the time it takes tap A on its own. How many hours would it take for tap C to fill the pool on its own?

   4. _____ hours

Bull's-eye, Page 2: Numbers and Combinatorics

5. What is the number which is halfway between $\frac{3}{4}$ and $\frac{4}{3}$? Express the answer as a common fraction.

5. _____

6. What common fraction between 0.91 and 0.97 has the least numerator?

6. _____

7. You start at corner A of equilateral triangle ABC with side 1 metre by taking a step to either B or C with probability $\frac{1}{2}$ each. You keep making such 1 metre steps, with probability $\frac{1}{2}$ to the corners you are not at. What is the probability of ending up back at A after taking exactly 4 steps? Express the answer as a common fraction.

7. _____

8. Betty and Ben each select independently and at random an integer between 0 and 5 (inclusive). What is the average non-negative difference between their numbers? Express the answer as a common fraction.
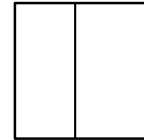
8. _____

Bull's-eye, Page 3: Geometry

9. The picture below shows a square and an equilateral triangle. If the degree measure of the angle labelled x is $34°$, what is the degree measure of the angle labelled y?
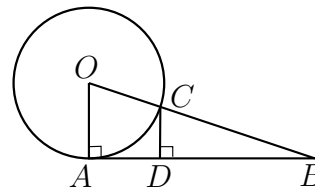
9. _____ degrees

10. A square is split into two rectangles as in the picture below. The smaller rectangle has area 8, and the larger one has area 10. What is the ratio of the perimeter of the smaller rectangle to the perimeter of the larger rectangle? Express the answer as a common fraction.
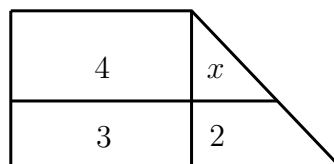
10. _____

11. In the picture below, the circle with centre O has radius 1. Point A lies on the circle, $\triangle OAB$ is right-angled at A, and $AB = 3$. The line segment OB meets the circle at C, and D on AB is such that CD is perpendicular to AB. Express the length of CD in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are integers, d is positive, no number greater than 1 divides all of a, b, and d, and no square greater than 1 divides c.

11. _____ units

12. In the picture below, lines that look perpendicular *are* perpendicular. The large trapezoid of the picture is divided into a trapezoid, two rectangles, and a triangle as shown. The trapezoid has area 2, and the rectangles have area 3 and 4 as shown. What is the value of x, the area of the small triangle? Express the answer as a common fraction.

12. _____ units^2

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. It so happens that $\sqrt{1800}+\sqrt{200}=\sqrt{n}$, where n is an integer. What is the value of n?

1. _____

2. The price of a commodity is adjusted upwards by 2.5% on January 15 of every year. What is the ratio of the price on January 16 of a certain year to the price on January 16 twenty years earlier? Provide the answer as a decimal correct to 2 decimal places.

2. _____

3. What is the area of the triangle whose vertices have coordinates $(0,0)$, $(5,7)$, and $(7,10)$? Express the answer as a common fraction.

3. _____ units^2

4. Dan had to pay \$2500 for an overseas school trip, and was charged simple yearly interest of 5% for late payment. If he was 15 days late, how much interest did he pay, in dollars, correct to 2 decimal places. Assume that there are 360 days in the year.

4. _____ dollars

5. Define the number N by

$$N = 123456789 + 234567891 + 345678912 + 456789123 + 567891234.$$

What is the sum of the digits of N?

5. _____

Co-op, Page 2: Team answers must be on the *coloured* page.
   Answers on a white page will not be graded.

6. How many integers a are there such that $1 \leq a \leq \sqrt{6400}$ and a divides 6400?

6. _____ integers

7. The world is divided into "rich," "emerging," and "poorest" countries. The people of the rich countries are asked to come to the rescue. The people of the poorest countries, who make up 53% of the world population, need $5000 per capita. The people of the emerging countries, who make up 36% of the world population, need $2000 per capita. If all the money is to come out of the pockets of each individual from the rich countries, how much will it cost each of them if the total population of the rich countries is 770 million? Give the answer rounded to the nearest dollar.

7. _____ dollars

8. It so happens that there are positive integers a, b, and c such that

$$\frac{355}{113} = a + \cfrac{1}{b + \cfrac{1}{c}}$$

What is the value of c?

8. _____

9. How many products of the form $a \times b \times c$ are there, if a, b, and c can be any of the primes 2, 3, 5, or 7? Note that $28 = 2 \times 2 \times 7$ is such a product (primes can repeat), and is to be counted as the same as $2 \times 7 \times 2$.

9. _____

10. What is the greatest integer n for which $\dfrac{24n}{n-4}$ is an integer?

10. _____

Co-op, Page 3: Team answers must be on the *coloured* page.
    Answers on a white page will not be graded.

11. What is the area, in square metres, of the smallest square that can be fully covered with no gaps or overlaps by using 50 cm by 50 cm tiles only, and also by using 40 cm by 60 cm tiles only.

11. _____ metres^2

12. The mean (average) of a and b is $3/4$ times the mean of a, b, and c. The mean of b and c is $4/3$ times the mean of a, b, and c. If a, b, and c are positive and the mean of a and c is k times the mean of a, b, and c, what is the value of k? Express the answer as a common fraction.

12. _____

13. How many ordered triples (i, j, k) of non-negative integers are there such that $i + j + k = 4$? Please note that $(4, 0, 0)$ is not the same as $(0, 4, 0)$.

13. _____ triples

14. In the game of *Lucky 7*, you roll a fair die a few times and try to reach a total sum of 7 on your rolls. There is only one rule: If on roll n you got a certain number k and on roll $n + 1$ you get a number equal to or larger than k, then the game is over after roll $n + 1$. A few valid sequences in the game are $(1, 1)$, $(1, 3)$, $(5, 4, 3, 3)$, $(6, 5, 5)$, $(6, 5, 4, 3, 2, 1, 5)$. Please note that the maximum number of rolls until the game is over is 7. What is the probability of reaching a total of 7 when the game is over? Two examples of winning sequences are $(1, 6)$ and $(2, 1, 4)$. Note that $(6, 1)$ is not a winning sequence since you still have to roll for a third time. Express the answer as a common fraction.

14. _____

15. The 5 students on the team that won the Provincial Math Challengers competition decided to celebrate the event with a gift exchange party. The rule is that each of the 5 students is to give one gift to *exactly* one *other* student. An example of such a gift exchange is "A gives to B, B gives to C, C gives to D, D gives to E, and E gives to A." How many ways are there to do the gift exchange?

15. _____ ways