Blitz, Page 1

- 1. Two fair dice are rolled. What is the probability that the sum of 1. the two numbers obtained is 8? Express the answer as a common fraction.
- 2. A square is inscribed in a circle of radius 11. What is the area of the 2. \_\_\_\_\_ units<sup>2</sup> square? 3. What is the integer nearest to  $(8.5)^2$ ? 3. 4. Express  $\frac{37}{91}$  as a decimal, correct to 2 decimal digits. 4. 5. Albert took the same algebra test a total of 3 times. Each time he 5. \_\_\_\_\_ questions took the test, the number of questions he answered correctly increased by 50%. If on the last test he got 36 of the 70 questions right, how many questions did he get right the first time he took the test? 6. Express  $\frac{3^3 - 3^2 - 3^1}{2^3 - 2^2 - 2^1}$  as a common fraction. 6. \_\_\_\_\_ 7. In 2009, Albert's wage was decreased by 20%. Recently, the resulting 7. \_\_\_\_\_ percent wage was increased by 20%. By how many percent is Albert's current

wage smaller than his pre-2009 wage?

1. \_\_\_\_\_

Blitz, Page 2

Blitz, Page 3

15. A regular pentagon is inscribed in a circle. You connect any two 15. corners with a straight line. Into how many regions is the circle divided?

- 16. If x and y are real numbers such that x + y = 11 and xy = 13, what 16. is the value of  $x^2 + y^2$ ?
- 17. Every interior angle of a many-sided regular polygon has measure 160 degrees. How many sides does the polygon have?
- 18. A loonie (1 dollar coin) weighs 4 times as much as a dime (10 cent 18. dollars coin). Bag A contains only loonies, bag B contains 5 times as many dimes as loonies, but no other coins or notes. Bags A and B have exactly the same weight. If bag A contains 45 dollars' worth of loonies, what is the value, in dollars, of the dimes in Bag B?
- 19. Let N be the smallest positive integer whose first 4 digits are 2, 0, 1, 0 (in that order) and which is divisible by 45. What is the value of  $\frac{N}{45}$ ?
- 20. How many numbers from 100 to 999 contain the digit 8 exactly once? 20. numbers



regions

19.\_\_\_\_\_

## Blitz, Page 4

- 21. A card is removed from a well-shuffled standard 52-card deck, and then a second card is removed. What is the probability the second card is of the same kind as the first card (so if the first card was a 7, the second should be a 7, if the first card was a King, the second card should be a King, and so on. Express the answer as a common fraction. Note that there are 4 cards of each kind.
- 22. You have 6 tables, whose tops are congruent equilateral triangles, 22. perimeters and have guests coming for dinner. You are allowed to join tables together into one or more groups, but if you do, edges of tables that are put together must match up, full edge to full edge. How many different perimeters can we obtained by combining tables in this way?
- 23. A *quadrilateral* is a closed *curve* made up of 4 line segments. (It does not include the "inside," and need not be convex.) If two quadrilaterals have finitely many points in common, what is the largest possible number of common points?
- 24. What is the smallest possible value of  $n^2 17n + 100$  as n ranges over the integers?
- 25. The rectangle below is made up of twelve  $1 \times 1$  squares. Three points, each of which is a vertex of a square, are chosen. Suppose these three points do not all lie on the same line. Form the triangle that has these three points as vertices. How many different numbers are there which could be the area of such a triangle?

26.

26. You throw a fair coin 6 times. The total number of heads you got is less than 5. What is the probability that the total number of heads is less than the total number of tails? Express the answer as a common fraction.

23. points

24.

25. numbers

21.



## Bull's-eye, Page 1: Problem Solving

1. A *rod* is 5.5 yards, and a *furlong* is 220 yards. How many rods are 1. \_\_\_\_\_ rods there in 6 furlongs?

2. For every 3 samosas bought at the full price of 40 cents for each 2. \_\_\_\_\_\_ samosas samosa, a store offers 2 samosas at half price. What is the largest number of samosas that Alicia can buy for less than \$9?

3.

3. In a college class of 60 people, there are 20 men and 40 women. Eighty percent of the men, and fifty percent of the women, are wearing jeans. If a jean wearer in the class is chosen at random, what is the probability that the chosen person is a woman? Express the answer as a common fraction.

4. Alicia and Beth differ in weight by 20 pounds. Beth and Gamal differ
4. \_\_\_\_\_\_ pounds in weight by 30 pounds. And Gamal and Delbert differ in weight by 6 pounds. What is the least possible weight difference between Alicia and Delbert?

## Bull's-eye, Page 2: Numbers and Combinatorics

5. What is the sum of all the *distinct* prime factors of 8888? (For 5. \_\_\_\_\_\_ example, the sum of the distinct prime factors of 12 is 5.)

6. Three dice are tossed, two red ones and a blue one. What is the probability that the number showing on the blue die matches at least one number showing on a red die? Express the answer as a common fraction.

- 7. At a restaurant buffet, there are four flavours of ice cream available: durian, lime, mango, and orange. In how many ways can Alan choose two scoops of ice cream? (Durian and lime is the same way as lime and durian, and Alan can choose two scoops of the same flavour.)
- 7. \_\_\_\_\_\_ ways

6.

8. What is the sum of all the positive factors of 2013? Note that 1 and 8. \_\_\_\_\_ 2013 are factors of 2013.

Bull's-eye, Page 3: Geometry

9. All the shapes that look like squares are squares. Each side of the largest square is 8 cm. What is the area, in cm<sup>2</sup>, of the shaded region?



11. A line has equation y = mx - 6. The line passes through the point (4, 10). If the x-ccordinate of a point on the line is 10, what is the y-coordinate of that point?

12. A cube with sides 1 is inscribed in a cone whose base has radius 1 in such a way that one face of the cube is in the base of the cone, and the other four vertices are on the curved surface of the cone. The volume of the cone is  $\pi x$ . Find x. Express the answer as  $r + s\sqrt{z}$ , where r and s are common fractions and z is an integer.



9. \_\_\_\_\_  $cm^2$ 

11. \_\_\_\_\_

12. \_\_\_\_\_

Co-op, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. Let 
$$A = \frac{3^2 - 2^2 - 2}{(\sqrt{2} \times 3^2 \sqrt{2}) - 2}$$
 and let  $B = \frac{3^2 + 15 - (\sqrt{100} + 5)}{2^2 + 2}$ .  
Express  $\frac{A}{B}$  as a common fraction.

- 2. The coordinates of three of the vertices of a rhombus are (20,0), (3,3), and (0,20). What is the sum of the x and y coordinates of the fourth vertex?
- 3. Let A be the set consisting of the numbers 1, 2, 3, 4, and 5. In symbols,  $A = \{1, 2, 3, 4, 5\}$ . Call a subset of A good if it contains two numbers that add up to 5. So  $\{1, 4\}$  is good, and  $\{1, 3, 4\}$  is also good. How many good subsets of A are there? Note that A is a subset of A.
- 4. You are allowed to use pennies, nickels, dimes, quarters, loonies (1\$) and toonies (2\$) to make any sum of money using two rules. (a) You are not allowed to use more than 4 pennies; (b) if you use more than 6 of one kind of coin, then you are not allowed to use more than three of any other kind. For example, you are allowed to use 7 toonies, 3 loonies, and one each of the other kinds, but you are not allowed to use 7 toonies, 4 nickels, and 3 each of the other kinds. What is the smallest amount of money that you cannot make? Express your answer in dollars correct to 2 decimal digits.
- 5. A rhombus ABCD has sides 1, and  $\angle DAB$  is 60°. A circle is tangent to line BC at B, and is tangent to the line segment AD at a point E between A and D.Find the area of the region which is inside the rhombus and also inside the circle. Express the area as a decimal, correct to 3 places after the decimal point.



1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_\_ subsets

4. \_\_\_\_\_

5. \_\_\_\_\_ units<sup>2</sup>

- Co-op, Page 2: Team answers must be on the *coloured* page. Answers on a white page will not be graded.
  - 6. A ball is dropped from a height of 20 metres. Each time it hits the 6. \_\_\_\_\_\_ times floor, it bounces to  $\frac{8}{9}$  of its previous height. How many times does the ball hit the floor until and including the first time it cannot reach a height of 1 metre?
  - 7. The basin of the Yukon River is 476237 km<sup>2</sup>. (That means that all 7. \_\_\_\_\_\_ the rainfall in that region is drained by that river.) The average yearly rainfall in the basin is 642 mm, and on average 13% of the rainfall is carried by the river to the ocean. How much water on average is carried by the river to the ocean in one day? Assume that there are 365 days per year. Express your answer in km<sup>3</sup>, correct to 3 places after the decimal point.
  - 8. In a certain day (24 hour period) the amount of water that was carried by the Yukon River to the ocean was  $1.200 \times 10^8$  m<sup>3</sup>. At the mouth of the river (the location where the river meets the ocean), the average width of the river was 250 m, and its average depth was 6.0 m. (For simplicity assume that the cross-section of the river at this location is rectangular.) Assume that the water is flowing at constant speed through that cross-section. What was the average speed of the water? Express your answer in m/sec, correct to 3 decimal digits.
  - 9. In stationary water, salmon swim (on average) at the speed of 1.2 m/sec as long as the temperature of the water is 10.0 degrees Celsius or less. For each 1 degree rise in water temperature after that, up to a temperature of 20 degrees, salmons' speed is reduced by a constant amount, in such a way that if the temperature is 20 degrees they cannot swim at all. Salmon can swim up the river for at most 20 days until they reach their spawning ground, and their swimming speed is reduced by the speed of the river flow. How far can they reach upriver if the water temperature is 12.0 degrees and if the river flows at 0.5 m/sec? Express your answer in km, rounded to the nearest km.
  - 10. You select an integer N from 1 to 100 inclusive, and calculate the sum from k = 1 to k = N of  $k^4$ , noting the units digit of that sum. What is the average value of these units digits? Express your answer correct to 1 digit after the decimal point. So for example an answer of 6.7 is of the right shape. Hint: Find the units digit for  $k = 1, 11, 21, \ldots$  and look for a pattern.

8. m/sec

 $\mathrm{km}^3$ 

9. \_\_\_\_\_ km

10.

- Co-op, Page 3: Team answers must be on the *coloured* page. Answers on a white page will not be graded.
  - 11. For any integer n > 0, the number T(n) is called the *n*-th triangular number if  $T(n) = 1 + 2 + 3 + \dots + n$ . What is the smallest positive integer n such that T(n) has exactly 16 positive factors? Note that 1 and m are factors of the positive integer m.
  - 12. If you toss a fair nickel 100000 times, it lands heads on average 49912 times, tails 49912 times, and on its edge the rest of the time. You threw the coin 5 times. What is the probability that it landed *exactly* 3 times on its edge. Express the answer in scientific notation, correct to 4 significant digits. An answer like  $2.047 \times 10^{-5}$  is of the right shape.
  - 13. A positive integer is called *square-free* if it is not divisible by any perfect square greater than 1. For example, 1, 2, and 6 are squarefree, while 4 and 18 are not. Two fair standard dice are tossed. What is the probability that the product of the two numbers obtained is square-free? Express the answer as a common fraction.
  - 14. A convex polygon has 2013 sides. Let N be the largest integer smaller 14. \_\_\_\_\_ polygons than 2013 such that N + 1 is prime. Using any N corners of this polygon as the corners of a new N-sided convex polygon, how many different N-sided convex polygons can be made? Hint: The correct answer is 10 digits long.
  - 15. You have a square with side 1, and select a point O at random inside the square, and draw a circle of radius 1 with O as its centre. What is the probability that the entire square is inside the circle? Express the answer as a decimal, correct to 3 decimal places.



15.

13.

12.

11.