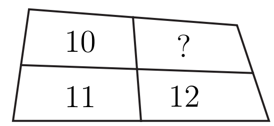
Problems, January 2006

Problem 1. A convex quadrilateral is split into four parts by joining the midpoints of opposite sides as in the picture below. If three of the parts, going counterclockwise, have area 10, 11, and 12, what is the area of the fourth part? (The picture is not drawn to scale.)



Problem 2. A triangle is isosceles. Suppose there is a line that divides the triangle into two isosceles triangles. What can we conclude about the angles of the original triangle?

Problem 3. Find the sum of all the four-digit numbers all of whose digits are odd.

Problem 4. Let $\mathcal{S}$ be a collection of 2006 points in the same plane.

(a) Show that there is a point P in $\mathcal{S}$, and a line passing through P, such that 500 of the points in $\mathcal{S}$ are on one side of the line and 1505 are on the other side.

(b) Suppose that no 3 points of $\mathcal{S}$ lie on the same line. Show that for *any* P in $\mathcal{S}$ there is a line through P such that 1002 of the points in $\mathcal{S}$ are on one side of the line and 1003 are on the other side.

Problem 5. You have 11 thin straight sticks. The length of each stick is a whole number of cm, and each stick has length 88 cm or less. Show that 3 of these sticks can be arranged to form a triangle. (Three positive numbers are the sides of a triangle if the sum of any two is greater than the third. The fact that 11 divides 88 has no bearing on the solution, I think.)

Problem 6. The school cafeteria offers three equally awful lunch choices A, B, and C. Every day, Zoe remembers how bad the immediately previous lunch was, and flips a fair coin to decide between the other two options.

On day 1 of school she had lunch A. Find the probability that she has lunch A on day 100.

`http://www.pims.math.ca/education/math_problems/`
`http://www.math.ubc.ca/~adler/problems/`