

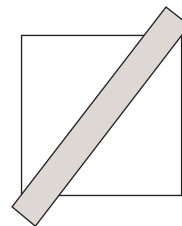
## Problems, January 2007

**Problem 1.** Find three numbers in geometric progression whose sum is 2 and the sum of whose squares is 8.

**Problem 2.** A tangent line to the ellipse  $x^2 + 4y^2 = 4$  meets the  $y$ -axis at the point  $(0, 2)$ . What is the point of tangency? No calculus please!

**Problem 3.** Thirteen people, including Alpha, Beta, Gamma, and Delta, are seated at random on the 13 chairs around a circular table. (a) What is the probability that no two of Alpha, Beta, and Gamma are immediate neighbours? (b) What is the probability that no two of Alpha, Beta, Gamma, and Delta are immediate neighbours?

**Problem 4.** A long ruler of width 1 is placed over an  $a \times a$  square sheet of paper so that one edge of the ruler passes through a corner of the square, and the other edge passes through the opposite corner. Find the fraction of the area of the paper which is covered by the ruler.



**Problem 5.** Note that the equation  $x^2 - 6y^2 = 1$  has  $x = 5$ ,  $y = 2$  as a solution. Find a solution  $(x, y)$  of this equation, where  $x$  and  $y$  are integers and  $x > 2007$ . Hint: Maybe rewrite the equation as  $(x - y\sqrt{6})(x + y\sqrt{6}) = 1$ .