

## Problems, January 2008

**Problem 1.** For any real number  $x$ , let  $\lfloor x \rfloor$  be the greatest integer which is less than or equal to  $x$ . It is easy to verify that  $\lfloor \sqrt{40} \rfloor = 6$ ,  $\lfloor \sqrt{4400} \rfloor = 66$ ,  $\lfloor \sqrt{444000} \rfloor = 666$ , and  $\lfloor \sqrt{44440000} \rfloor = 6666$ . State and prove the general result that these computations suggest.

**Problem 2.** Simplify  $\left(\cos \frac{\theta}{2}\right) \left(\cos \frac{\theta}{4}\right) \left(\cos \frac{\theta}{8}\right) \cdots \left(\cos \frac{\theta}{512}\right) \left(\cos \frac{\theta}{1024}\right)$ .

**Problem 3.** The figure below is a regular hexagon of area 1. Find the area of the shaded region.



**Problem 4.** Thirty students each took pass/fail tests #1 and #2. At least one doubly unfortunate student failed both. Find a simple *expression* for the number of different ways this could have happened. (Here, *which* individuals pass which test(s) matters.)

**Problem 5.** Find all configurations of 4 distinct points in the plane such that exactly two different numbers occur as distances between pairs of these points. (One such configuration consists of the 4 vertices of a square. The two different distances are the length of an edge and the length of a diagonal.)