Problems, January 2012

Problem 1. Alan and Beti alternately toss a fair die, with Alan going first. A neutral third party keeps a running tab of the combined sum of all their throws. Whoever first reaches a combined sum divisible by 6 wins. What is the probability that Alan wins?

Problem 2. Triangle ABC is isosceles, and right-angled at C. Point P is 3 units from A and 4 units from B. What is the largest possible distance from P to C?

Problem 3. A regular 49-gon $\mathcal{G}$ is inscribed in a circle. How many of the triangles whose vertices are vertices of $\mathcal{G}$ have the centre of the circle in their interior?

Problem 4. The positive integer n is called a *perfect power* if there exist integers a and b, with $b > 1$, such that $n = a^b$. For what primes p is $3^p + 4^p$ a perfect power?

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/