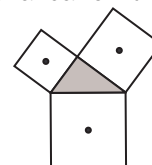


## Problems, October 2007

**Problem 1.** Find all real numbers  $x$  such that

$$\sqrt[3]{25+x} + \sqrt[3]{25-x} = 5.$$

**Problem 2.** Outward facing squares are erected on the sides of a right-angled triangle whose legs have length  $a$  and  $b$ . What is the area of the triangle whose vertices are the midpoints of these squares?



**Problem 3.** A sequence  $a, b, c$  of non-zero real numbers is called a (three-term) *geometric sequence* if  $b/a = c/b$ . Find all three-term geometric sequences  $a, b, c$  such that  $a, b$ , and  $c$  are integers and  $a + b + c = 39$ . There is a high probability of missing some sequence(s), so you should *prove* that you got them all—not being able to think of more is not a proof.

**Problem 4.** Let  $\alpha, \beta$ , and  $\gamma$  be the angles of a triangle. Can  $\tan \alpha, \tan \beta$ , and  $\tan \gamma$  all be integers?

**Problem 5.** For any non-negative integer  $n$ , let  $a_n$  be the remainder when  $2^n$  is divided by 10000. Show that the sequence  $a_0, a_1, a_2$ , and so on is ultimately periodic.