

Problems, October 2009

Problem 1. Alphonse and Beth are mathematicians and partners. In their absence, a third person (Gamay) places the King of Spades, the Queen of Hearts, and the 2 of Clubs face down, *in a row*, in random order.

Alphonse enters the room and turns one of the cards face up. Then he turns another card face up. If one of the cards Alphonse turned face up is the King of Spades, Gamay gives Alphonse \$1.

Gamay then turns Alphonse's two cards face down, and Beth enters the room. She turns one of the cards face up. Then she turns another card face up. If one of the cards Beth turned face up was the Queen of Hearts, Gamay gives Beth \$1. If Alphonse and Beth together use best strategy, what is the probability they get a combined total of \$2?

Problem 2. Let $f(x, y) = x^2 - 2xy + 3y^2 - 4x + 5y$. What is the smallest value of $f(x, y)$ as x and y range independently over the real numbers? (Proof is needed that the value given indeed is the smallest.)

Problem 3. The altitudes from two vertices of a triangle have length 3 and 6 respectively. What are the possible values of the length of the altitude from the third vertex? (Proof is needed that the claimed possible values are indeed possible, and that no others are.)

Problem 4. Define the Fibonacci sequence F_0, F_1, F_2 , and so on by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 0$. Let m be a positive integer. Show that there is a positive integer $N = N(m)$ such that m divides F_N .