

Problems, October 2010

Problem 1. Find (with proof) all primes p such that $p+6$, $p+12$, $p+18$, and $p+24$ are all prime.

Problem 2. Consider the system of equations $x + y + z = a$, $xy + bz = 6$. For what integer values of a and b does the system have infinitely many integer solutions (x, y, z) ?

Problem 3. (a) Among all triangles that contain a 1×1 square, what is the smallest possible area? (b) And among these triangles that contain a 1×1 square and have smallest possible area, what is the smallest possible perimeter?

Problem 4. Without using a calculator, express $2 \times 10^5 - \sqrt{10^{10} - 1} - \sqrt{10^{10} + 1}$ in “scientific” notation, correct to 2 significant figures. (You may want to use a calculator to help you guess what the answer might be.)