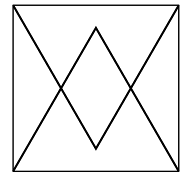
Problems, October 2011



Problem 1. Two equilateral triangles are erected on opposite sides of a 1×1 square as shown. Find an exact expression for the area of the region that is common to these two triangles.

Problem 2. Find (with proof) the product of all the real solutions of the equation

$$x^{101} - 4x^{99} + x^{98} - 4x^{96} + x^{95} - 4x^{93} + \cdots + x^5 - 4x^3 + x^2 - 4 = 0.$$

Problem 3 (Modified)**.** Find, with proof, the number of ways that 36 can be represented as a sum of one or more positive even integers. Here the order of summation matters. So for example $36 = 6 + 6 + 20 + 4$ is to be counted as different from $36 = 20 + 6 + 6 + 4$. (The original question asked for sum of one or more *odd* integers, and is too hard.)

Problem 4. Prove that the only integer solution of the equation

$$x^2 + y^2 + z^2 + w^2 = 8(xy + yz + xz)$$

is given by $x = y = z = w = 0$.

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/