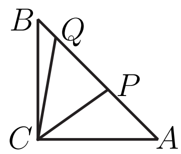
Problems, November 2008

Problem 1. Let D be the 10-element set consisting of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. How many ordered pairs (A, B) are there such that the union of the sets A and B is equal to D? One such ordered pair has $A = \{0, 1, 2, 5, 6, 8, 9\}$ and $B = \{0, 1, 3, 4, 7, 9\}$.

Problem 2. Let ABC be an isosceles triangle which is right-angled at C. Let P and Q be points on the hypotenuse AB, with P and Q coming in the order shown in the picture below. Suppose that $\angle QCP$ has measure 45°. Show that $(AP)^2 + (BQ)^2 = (PQ)^2$.



Problem 3. Alphonse and Beti are mathematicians who collaborate in separating people from their money. Alphonse is blindfolded. The mark (whose name is Mark) is asked to remove 5 cards from a standard 52-card deck, and hand them back to Beti. Beti gives one of the 5 cards to Mark (keeping 4). Mark puts that card back in the deck, and shuffles thoroughly. Beti then arranges the remaining 4 cards in a neat face-up row. Beti gives 10 to 1 odds that Alphonse can find the card that Mark had put in the deck. Alphonse takes off the blindfold, looks at the 4 cards in the row Beti made, and identifies the missing card. How can this be done?

Problem 4. Find all ordered triples (a, b, c) of positive integers such that b divides $2a + 1$, c divides $2b + 1$, and a divides $2c + 1$. (A correct list is not enough: one must show that the list is complete.)

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/