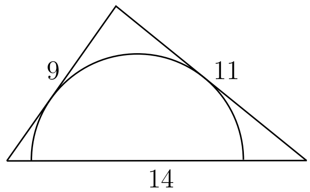
Problems, November 2009

Problem 1. A triangle has sides 9, 11, and 14. A semicircle is inscribed in this triangle, with diameter on the side of length 14. Give an exact expression for the radius of the semicircle.



Problem 2. The numbers 1, 2, 3, $\dots$, 2009 are written on index cards, one to a card. The cards are laid out in a row, in some order. Now do the following operation over and over again. Look at the leftmost card: if k is the number written on it, reverse the order of the first k cards, and leave the others where they are. For example, suppose we are looking at the numbers 1 to 9 instead of 1 to 2009, and our initial order is 395672814. Then we next get 593672814 (the first 3 cards have been reversed), and then 763952814 (the first 5 cards have been reversed), and so on.

Show that, after a while, the leftmost card has 1 written on it, so that after a while the order of the cards does not change.

Problem 3. Show that $2^{99} + 3^{99}$ is divisible by 35.

Problem 4. Find, with proof, the largest possible value of the product $(x_1)(x_2)(x_3)\cdots(x_n)$, as n, and x_1, x_2, x_3, $\dots$, x_n range over all positive integers such that

$$x_1 + x_2 + x_3 + \cdots + x_n = 2009.$$

(Note that n is at our disposal, we are free to have as many x_i as we wish, up to 2009.)

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/