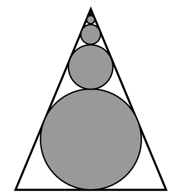
Problems, November 2010

Problem 1. The base of an isosceles triangle is 20, and the equal sides are both 26. There are infinitely many shaded circles. The largest is inscribed in the triangle, and each of the others is tangent to two sides of the triangle and to the circle below it. What is the sum of the perimeters of *all* the circles?



Problem 2. Define the sequence A_0, A_1, A_2, and so on by $A_0 = A_1 = 1$, and $A_n = 2A_{n-1} + A_{n-2}$ for $n \geq 2$. Let $x = 1/3$. Calculate

$$A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots + A_n x^n + \cdots .$$

Manipulate "infinite sums" freely, assume they behave algebraically like finite sums.

Problem 3. Without using calculus, find the slope of the tangent to the hyperbola $xy = 1$ at the point $(1/3, 3)$. Ideally, find two entirely different approaches.

Problem 4. Beth has a biased loonie that lands heads with probability p, and tails with probability $1 - p$. Alicia tosses the coin repeatedly, and keeps a running count of heads and tails. If the number of heads is *ever* greater than the number of tails, Alicia wins the game (and the coin). What is the probability that Alicia wins the game? There are sharp differences between the cases $p < 1/2$ and $p \geq 1/2$.

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/