Problems, November 2011

Problem 1. As a classroom assignment to the Grade 4 class, the teacher asked them to find the sum of the decimal digits of all the numbers from 1 to 2011, inclusive. Little Carlotta found the answer in a couple of minutes. What is that answer?

Problem 2. Let $N(r)$ be the number of points (a, b) with integer coordinates inside or on the circle with center the origin and radius r. Show that there is a positive real number R such that $3.14R^2 < N(R) < 3.15R^2$.

Problem 3. Find exact expressions for all the real roots of the equation

$$2\sqrt[3]{2x-1} = x^3 + 1.$$

Problem 4. Let $f(x) = (10 - x)(10 - \sqrt{9^2 - x^2})$. Find the largest value and the smallest value taken on by $f(x)$ as x ranges over the interval $0 \leq x \leq 9$. Ideally, the argument should not involve the calculus.

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/