Problems, December 2006

Problem 1. Alphonse ran in a cross-country race, running half of the *distance* at 3 minutes per km and half at 3 minutes 10 seconds per km. If he had run half of the *time* at 3 minutes per km, and half at 3 minutes 10 seconds per km, it would have taken him 1 second less to finish the race. How long did Alphonse actually take?

Problem 2. Imagine calculating the decimal expansion of $(10 + \sqrt{101})^{99}$. What are the first two *non-zero* digits after the decimal point?

Problem 3. There are six tickets in a box, with the numbers 1 to 6 written on them. The tickets are taken out of the box one at a time. In how many different ways can this be done, if at any stage of the process the numbers already taken out have to be a *set* of consecutive integers, not necessarily in their natural order? One of these ways is withdrawal in the order 435621, for note that each of $\{4\}$, $\{4, 3\}$, $\{4, 3, 5\}$, $\{4, 3, 5, 6\}$, $\{4, 3, 5, 6, 2\}$, and $\{4, 3, 5, 6, 2, 1\}$ is a set of consecutive integers. It would be best to take an approach that generalizes readily to the numbers from 1 to n.

Problem 4. Two circles have the same center; one has radius 2 and the other has radius 3. Points P and Q are chosen independently and at random on the boundary of the outer circle. Find, correct to 3 decimal places, the probability that the line PQ passes through the inner circle.

Problem 5. Find all integers n such that $2^n + n^2$ is a perfect square, and show that you have found them all.

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/