

Problems, December 2007

Problem 1. Which straight lines have exactly one point in common with the circle with equation $x^2 + y^2 = 1$, and have exactly one point in common with the parabola $y = x^2 + 1$?

Problem 2. Find distinct integers m and n such that $2^n - 2^m$ is a multiple of 2008.

Problem 3. What is the largest possible common divisor of $n^3 + 1$ and $n^2 - 5$, as n ranges over the integers?

Problem 4. Let w be the number of “words,” all of whose letters are different, over the standard 26-letter alphabet. Examples of such words are w , bza , and $kzlfhg$ —there are much longer ones. Find $w/26!$, correct to 2 decimal places.

Problem 5. Let n be a positive integer. Which is bigger, $999^n + 1000^n$ or 1001^n ? Give as detailed an analysis as possible.