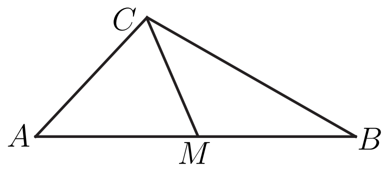
Problems, December 2008

Problem 1. In the figure below, $AB = 4$, $BC = 3$, $AC = 2$, and M is the midpoint of the line segment AB. What is the length of the median CM?



Problem 2. Prove, using only "high school" ideas, that for any positive integer n,

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots + \frac{1}{n^2} < \frac{5}{3}.$$

(It turns out that the infinite sum $1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ is equal to $\pi^2/6$. The desired inequality then follows from standard estimates of π. But that approach would take us well beyond high school ideas.)

Problem 3. Find a simple expression for

$$\binom{100}{1} + 2\binom{100}{2} + 3\binom{100}{3} + \cdots + 99\binom{100}{99} + 100\binom{100}{100}.$$

(Here, $\binom{n}{r}$ denotes the number of ways of choosing r objects from n objects. On scientific calculators, $\binom{n}{r}$ is usually denoted by ${}_n\mathrm{C}_r$.)

Problem 4. (a) Is there a non-constant polynomial $P(x)$ such that whenever a is rational, $P(a)$ is irrational?

(b) Is there a non-constant polynomial $P(x)$ such that whenever a is irrational, $P(a)$ is rational?

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/