

Problems, December 2010

Problem 1. A square is cut up into 169 squares. At least 168 of these are 1×1 squares. What are the possible side lengths of the 169-th square?

Problem 2. Let $f(x) = (x^2 + x + 1)/(x^2 - x + 1)$. What are the possible values of $f(x)$, as x travels over the real numbers? A graphing program or calculator may lead to a plausible conjecture, but proof is needed that the conjecture is correct. No calculus, please!

Problem 3. Imagine tossing a fair coin over and over again, until you get three consecutive heads. What is the mean of the total number of tosses required?

Problem 4. Let S be any infinite set of positive integers such that no element of S has a prime divisor greater than 7. Show that S has an infinite subset C such that if a and b are in C , then a divides b or b divides a .