

Problems, December 2011

Problem 1. How many factors of 111^{2011} have last digit 1?

Problem 2. Find (with proof) all values taken on by $\frac{x}{x^2 - 4x + 3}$ as x ranges over the real numbers.

Problem 3. Let $P(n)$ be the product

$$\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{2^3}\right) \cdots \left(1 - \frac{1}{2^n}\right).$$

Find, with proof, a number $c > 0$ such that $P(n) > c$ for all n . Please note that the c need not be the largest c that will work, nor anywhere near it. Wolfram Alpha says that $P(n)$ is always bigger than 0.288788.

Problem 4. Find all triples (x, y, z) of non-negative integers such that $x \leq y \leq z$ and $xyz = 2(x + y + z)$. Proof is needed that the list is complete.