Problems, February 2007

Problem 1. Let $M = (1/2, 1/2)$. Find points A and B on the curve $y = 1/x^2$ such that M is the midpoint of the segment AB.

Problem 2. Simplify $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + 99 \cdot 100$. (We could find the answer by doing a long calculation, or by writing a program to do it. That's not what is wanted here.)

Problem 3. (a) Show how to divide the set $\{1, 2, 3, 4, \ldots, 98, 99\}$ into three subsets (not necessarily of the same size) so that the sum of the numbers in all three sets will be the same. (b) For what positive integers n can the same task be accomplished with the set $\{1, 2, 3, 4, \ldots, n-1, n\}$?

Problem 4. Find (with proof) the smallest possible value of $xy + yz + xz$, given that x, y, and z are real numbers such that $x^2 + y^2 + z^2 = 1$.

Problem 5. Let $N = (3 + 2\sqrt{2})^{512} + (3 - 2\sqrt{2})^{512}$. What is the rightmost decimal digit of N?

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/