

## Problems, February 2008

**Problem 1.** An Egyptian-style pyramid has a square  $b \times b$  base and height  $h$ . What is the volume of the smallest sphere that contains the pyramid? (This may be a little trickier than it looks.)

**Problem 2.** For any positive integer  $n$ , let  $f(n)$  be the integer nearest to  $\sqrt{n}$ . Evaluate

$$\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \cdots + \frac{1}{f(9999)} + \frac{1}{f(10000)}.$$

**Problem 3.** Show that the equation  $x^3 + 2y^3 + 4z^3 = 0$  has exactly one solution in integers.

**Problem 4.** Let  $S$  be a set of 51 integers chosen from  $\{0, 1, 2, \dots, 99\}$ . Show that there are two integers in  $S$  which differ by exactly 10.

**Problem 5.** A fair die is tossed  $n$  times. Find a *simple* expression for the probability that the number of 6's obtained is even.