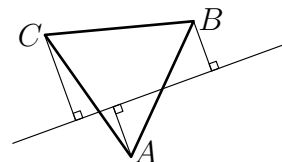


## Problems, February 2009

**Problem 1.** Vertex  $A$  of an equilateral triangle lies on one side of a certain line, and is at distance 2 from that line. Vertices  $B$  and  $C$  of the triangle are on the other side of the line, and, respectively, at distances 2 and 3 from that line. What is the length of a side of the triangle? (The diagram is not drawn to scale.)



**Problem 2.** We have an unlimited number of 1 gram weights, 10 gram weights, 100 gram weights, and 1000 gram weights. How many different combinations of weights is it possible to put in one pan of a pan balance so as to balance a 2009 gram kitten in the other pan?

**Problem 3.** Show that if  $x \geq 1$ , then

$$\sqrt{x+2} - \sqrt{x} < \frac{1}{\sqrt{x}} < \sqrt{x+1} - \sqrt{x-1}.$$

**Problem 4.** (a) Show that if the positive integer  $n$  is a multiple of 3, then  $7^n - 6^n$  is a multiple of 127. (b) Show that if  $n$  is not a multiple of 3, then  $7^n - 6^n$  is not a multiple of 127.