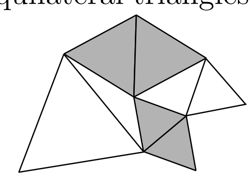
# Problems, February 2010

**Problem 1.** Every point on the circumference of a circle is coloured red or blue. Show that there is an isosceles triangle whose vertices lie on the circle and are all of the same colour.

**Problem 2.** The picture shows six equilateral triangles, and two triangles that don't look equilateral. Two pairs of congruent equilateral triangles are joined to form the shaded rhombi. Show that the sum of the areas of the rhombi is equal to the sum of the areas of the unshaded equilateral triangles.



**Problem 3.** For any real number $x$, let $\lfloor x \rfloor$ be the greatest integer which is less than or equal to $x$. For example, $\lfloor \pi \rfloor = 3$. Show that if $n$ is a non-negative integer, then

$$\left\lfloor \sqrt{n} + \sqrt{n+1} \right\rfloor = \left\lfloor \sqrt{4n+2} \right\rfloor .$$

**Problem 4.** For any positive integer $n$, define $n!!$ by

$$n!! = \begin{cases} (n)(n-2)(n-4)\cdots(1) & \text{if } n \text{ is odd and} \\ (n)(n-2)(n-4)\cdots(2) & \text{if } n \text{ is even.} \end{cases}$$

Show that $2009!! + 2010!!$ is divisible by 2011.