

Problems, February 2012

Problem 1. A certain curve consist of all points (u, v) such that there are two tangent lines to the curve $y = x^2$ that pass through (u, v) , and these tangent lines are perpendicular to each other. Find an equation for the curve.

Problem 2. How many different 12-letter "words" can be made using 3 A's, 4 B's, and 5 C's which have no consecutive occurrences of the letter B?

Problem 3. Find (with proof) all quadruples of consecutive odd integers whose product is a perfect square.

Problem 4. Suppose there is a number M such that for all n ,

$$|f(x_1 + x_2 + \cdots + x_n) - f(x_1) - f(x_2) - \cdots - f(x_n)| < M.$$

Show that $f(x + y) = f(x) + f(y)$ for all real numbers x and y .