

Problems, March 2007

Problem 1. Show how to divide any triangle \mathcal{T} into 2007 triangles similar to \mathcal{T} .

Problem 2. Find all x that satisfy the inequality

$$\left(\frac{\sqrt{x+1}-1}{x}\right)^2 > x+1.$$

Problem 3. Download speed varies. Your browser posts a running estimate of the time needed to finish the download job. It does so by assuming that the *average* download speed so far will continue. The browser was downloading a large file. After 2 minutes, it estimated that there were 30 seconds left. For the next 5 minutes, the estimated remaining time to finish the job stayed at 30 seconds. What fraction of the file had been downloaded after these 5 minutes? (Assume that the program continued to function normally.)

Problem 4. Find the maximum value taken on by $3x + 4y$ as (x, y) ranges over all ordered pairs such that $x^2 + y^2 \leq 1$.

Problem 5. The number 2^{10} is “nearly” 10^3 , in the sense that $2^{10}/10^3$ is close to 1. Show that there are positive integers a and b such that

$$0.998 < \frac{2^a}{10^b} < 1.002.$$

(Logarithms may be useful but are not essential.)